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DOI: <https://doi.org/10.1016/j.jedc.2015.02.013>

Posted at the Zurich Open Repository and Archive, University of Zurich

ZORA URL: <https://doi.org/10.5167/uzh-109871>

Journal Article

Accepted Version

Originally published at:

Röhrs, Sigrid; Winter, Christoph (2015). Public versus private provision of liquidity: is there a trade-off? *Journal of Economic Dynamics and Control*, 53:314-339.

DOI: <https://doi.org/10.1016/j.jedc.2015.02.013>

Public versus Private Provision of Liquidity: Is There a Trade-Off?

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This Version: January 2015

Abstract

To what extent is public debt private liquidity? Much policy advice given in the aftermath of the financial crisis rests on the assumption that increasing public debt relaxes borrowing constraints of private households. This is the case for ad-hoc debt limits, which are exogenous to public policy. Instead, if debt limits are fully endogenous, as e.g. in the case of the natural borrowing limit, public debt has no impact. We assume that borrowing limits arise because of limited contract enforceability and are therefore determined as equilibrium outcomes. Using an incomplete markets economy in which households are subject to uninsurable earnings shocks, we show that public debt provides some liquidity, but less so than it would if constraints were imposed ad-hoc. We show that generating borrowing constraints as an equilibrium outcome substantially alters the answers to other important questions, such as for the welfare effects of government debt or its impact on real economic activity.

Key words: Government Debt, Equilibrium Borrowing Constraints, Limited Commitment, Incomplete Markets, Crowding Out

JEL classification: E2, E62, E44, D52,

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1 Introduction

To what extent is public debt private liquidity? Put differently, to what extent does public debt relax the borrowing constraints of private households? We show that the answer to this question depends on the way these constraints are modeled.

Two cases have mainly been discussed in the literature. If households face an ad-hoc debt limit, which is exogenous to public policy, the government can, under certain assumptions, in effect fully relax the constraint by issuing bonds. This interaction between public debt and private borrowing constraints has been emphasized by Woodford (1990), Aiyagari and McGrattan (1998), Flodén (2001), Shin (2006) and Azzimonti, de Francisco, and Quadrini (2014), among others. The finding that public debt can relax private borrowing limits is also at the core of proposals aiming at raising government debt to offset the tightening of borrowing constraints households experienced during the financial crisis. See e.g. the recent work by Guerrieri and Lorenzoni (2011) and Eggertson and Krugman (2012).

If instead the debt limit is given by the natural borrowing constraint, which states that households can borrow up to the present value of their future (minimum) lifetime labor income, an increase in public debt tightens the debt limit, to the extent that the increase in public debt decreases the present value of households' future lifetime labor income. As a consequence, government in effect cannot relax households' debt limit. See e.g. Eggertson and Krugman (2012).

These two specifications of the borrowing constraint are silent about the precise microfoundation that causes the borrowing limit in the first place. In reality, debt constraints are likely to arise because of agency problems (e.g. adverse selection, limited commitment) or other type of frictions (e.g. transactions costs) in credit markets.

Ignoring the specific nature of the friction when studying the impact of public policy might be problematic, as public policy may affect the size of the friction and therefore also the borrowing constraint. This point was first made by Hayashi (1985) and Yotsuzuka (1987), who discuss several agency problems and conclude that the degree to which government debt has real effects depends on the specific nature of the problem. Real effects are strongest if government debt can substitute for missing private credit. However, if the agency problem is such that government debt merely replaces private credit, Ricardian equivalence (see Barro 1974) will continue to hold, even if there are liquidity constraints, and government debt has no real effects.

Our contribution is to study government debt in an environment in which borrowing limits emerge because private debt contracts are not enforceable. Using this environment, we aim at answering the following questions. How does public debt affect the provision of private liquidity (i.e. credit)? What does this imply for the effects of government debt on real activity, in particular the accumulation of private capital and the equilibrium prices of capital and labor? And, finally, how does the interaction between public debt and the provision of private credit influences the welfare effects of government debt?

The limited commitment environment is embedded in a production economy in which markets are incomplete, as in Aiyagari (1994). Households are subject to idiosyncratic income realizations. There is no aggregate risk, implying that government debt and private capital are perfect substitutes from the point of view of households who wish to transfer resources across periods.¹ Households can borrow

¹Gomes, Michaelides, and Polkovnichenko (2008) study an economy with aggregate risk, in which government debt and private capital are imperfect substitutes. They assume ad-hoc borrowing limits.

and lend using an asset which pays off independent of the realization of the idiosyncratic income shock. Following Zhang (1997), Alvarez and Jermann (2000), Kehoe and Levine (2001), Krueger and Perri (2011) and Ábrahám and Cárceles-Poveda (2010), borrowers can default on their debt obligations. If they do so, they are excluded from borrowing and lending forever. Borrowing limits are set in equilibrium such that borrowers always have an incentive to repay their debt, independently of the realization of their income process. The resulting borrowing limits are tighter than the natural borrowing limit, which we define by the present value of the minimum lifetime labor income, following Aiyagari (1994), but looser than the popular ad-hoc limit of zero, which restricts private borrowing altogether. To the extent that the provision of government debt affects the incentive to default, it will also affect the borrowing limit.

Assuming market incompleteness is appealing in our context since it allows us to generate a realistic wealth distribution (see Cordoba 2008). This is because market incompleteness limits risk-sharing opportunities. A realistic degree of wealth inequality is important for our purpose, since a large fraction of US households are in debt and therefore strongly affected by changes in the borrowing constraint. Moreover, the extremely unequal asset distribution observed in the US implies that a large fraction of the population receives income mainly from supplying labor. This, in turn, is important in order to evaluate the welfare effects of government debt arising from the changes in the equilibrium prices for capital and labor, which have a different impact on the wealth-rich and the wealth-poor (see Röhrs and Winter 2014). Moreover, Ábrahám and Cárceles-Poveda (2010) show that if markets are incomplete, assuming limited commitment implies that borrowing limits are monotonically increasing in income, a feature that is consistent with the data.²

Because the equilibrium borrowing constraints are tighter than the natural debt limit, Ricardian equivalence does not hold. A higher public debt/GDP ratio therefore crowds out private capital, and the equilibrium interest rate rises. Laubach (2009) empirically documents that an increase in the government debt/GDP ratio has indeed a significant positive impact on the real interest rate in the US. A higher interest rate, in turn, makes it more attractive for debtors to renege on their obligations. By comparing stationary equilibria, we illustrate that credit limits set by private lenders are increasing in the stationary government debt/GDP ratio. Our framework therefore suggests that there is indeed a trade-off between public debt and the supply of private credit. Interestingly, this trade-off arises because changes in government debt affect aggregate prices. Hence, an additional contribution of our paper is to highlight the importance of general equilibrium effects, which are not captured by the partial equilibrium models presented in Hayashi (1985) and Yotsuzuka (1987).

Moreover, we show that the response of the equilibrium debt constraint to a change in public debt is in between the two polar cases ad-hoc debt limit on the one hand and natural borrowing constraint on the other hand. More precisely, the equilibrium debt limit responds by less than the natural borrowing constraint, but more than an ad-hoc debt limit.

Our finding has important consequences for the effective bindingness of the constraint. Due to the endogenous response of the equilibrium constraint, government debt can only partially relax the debt limit for private households. It is therefore important to be explicit about the microfoundations

²Broer (2013) argues that an incomplete markets model in the tradition of Aiyagari (1994) is more in line with the joint distribution of income, consumption and wealth in the US, relative to a limited commitment model with complete markets.

underlying the private borrowing constraint. Reduced-form specifications such as an ad-hoc limit or the natural borrowing constraints (or an arbitrary convex combination of the two) are likely to lead to wrong conclusions regarding the link between public debt and private borrowing constraints.

Our quantitative framework also allows us to study the impact of equilibrium borrowing constraints on the welfare consequences of government debt. We focus on comparing stationary equilibria, following Aiyagari and McGrattan (1998) and Flodén (2001). Relative to the case in which borrowing limits are exogenously kept at their level corresponding to the long-run average debt/GDP in the US of 66 percent, the interest rate and the wage rate react less to a decrease in the debt/GDP ratio, and more to an increase in the debt/GDP ratio, relative to the long-run average. The different reaction of aggregate prices is driven by the endogenous response of the equilibrium borrowing limit. As a consequence, the long-run welfare effects of changes in the debt/GDP ratio are significantly smaller when borrowing limits are an equilibrium outcome. The gap can be as large as 0.45 percentage points of lifetime consumption of the average household. Our results therefore highlight the importance of borrowing constraints for the welfare effects of government debt.³

Our work contributes to several strands of literature. The framework is closely related to the work by Ábrahám and Cárceles-Poveda (2010) who study a revenue neutral tax reform that eliminates capital income taxation. Since Ábrahám and Cárceles-Poveda (2010) abstract from government debt, they cannot analyze the trade-off between public debt and private credit, which is the main focus of our paper. The interactions between public and private insurance in models with limited commitment are also studied in other papers. Attanasio and Ríos-Rull (2000) analyze how certain types of insurance for aggregate shocks affect private allocations. Krueger and Perri (2011) as well as Broer (2011) asks whether the government can provide public insurance against idiosyncratic income risk by implementing a progressive tax system. In line with these papers, our results also suggest that the provision of public insurance crowds out private insurance.

Our paper is also related to the strand of literature that analyzes the role of public debt in relaxing liquidity constraints in the production side of the economy, following the seminal work by Holmstrom and Tirole (1998). In many papers, borrowing constraints take the form of collateral constraints. A recent example is the work by Angeletos et al. (2012).

In these models, more government bonds mean more collateral and therefore also more intermediation of private credit. This implication is different from our model, where increasing public debt leads to tighter borrowing limits and therefore less private credit, even though more bonds still imply that effective constraints become less binding. Note that we focus on uncollateralized credit of households, in the spirit of Woodford (1990), Aiyagari and McGrattan (1998), Flodén (2001) or more recently Guerrieri and Lorenzoni (2011) and many others. We abstract from collateralized credit, because households typically use durable goods such as cars or houses to secure their loans, (see for example Fernández-Villaverde and Krueger 2011), and not government bonds.

We also contribute to the debate on the importance of generating borrowing limits as an equilibrium outcome for the analysis of public policy. Mateos-Planas and Seccia (2006) study a change in social

³In general, however, we find that the debt/GDP ratio that maximizes stationary welfare is negative, i.e. the government should hold assets, not debt, implying that the government should not fully relax private borrowing constraints. See Yared (2013) for a similar finding. In our case, the overall welfare effects are driven by the development of equilibrium prices, as in Flodén (2001) or Röhrs and Winter (2014).

security that reduces income variability in an exchange economy with incomplete markets. They find that equilibrium credit limits have almost no impact on the aggregate welfare effects of social security if the economy is closed. Andolfatto and Gervais (2008) as well as Rojas and Urrutia (2008) analyze the impact of social security in a life cycle model with incomplete markets. They conclude that social insurance has different welfare and distributional implications under equilibrium debt constraints. We find that equilibrium borrowing constraints substantially alter the welfare effects of government debt.

The remainder of the paper is structured as follows: In section 2 we describe a simplified version of our model, which allows us to derive analytical results. Section 3 introduces the quantitative model. We discuss its calibration in Section 4. Section 5 contains a discussion of our results. Finally, Section 6 concludes and contains suggestions for further research.

2 Government Debt and Private Liquidity: Analytical Results

The aim of this project is to study the link between government debt and private borrowing limits in a quantitative model. In this section, we discuss a simplified version of our quantitative model which allows us to identify the main channels through which government bonds and private debt constraints interact. In this simplified economy, we can replicate two well-known results. Under ad-hoc borrowing limits, which are exogenous to public debt, there is a one-to-one relationship between government debt and the effective private debt limit, i.e. the private borrowing limit net of government debt. Therefore, an increase in public debt makes the borrowing constraint less binding for private households, see Woodford (1990) and Aiyagari and McGrattan (1998).

The natural borrowing limit, on the other hand, is directly affected by an increase in public debt, such that there is no impact on the effective debt limit, see e.g. Eggertson and Krugman (2012).

Our contribution is the analysis of a third case, in which the private borrowing limit is endogenously generated in equilibrium by assuming limited commitment. We show that in this case, the response of the private borrowing limit is less pronounced than under the natural debt limit, but stronger than under the ad-hoc constraint.

2.1 Economy

We consider a neoclassical growth model with incomplete markets where households face uninsurable income shocks, as in Aiyagari (1994). The economy consists of three sectors: households, firms and a government. In the following, we describe the three sectors in greater detail.

Households. The economy consists of a continuum of ex-ante identical, infinitely lived households with total mass of one. Households maximize their expected utility by making a series of consumption and saving choices subject to a budget constraint and a borrowing limit on assets. In period $t = 0$, before any uncertainty has been realized, their expected utility is given by

$$U(c) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \quad (1)$$

where $c \equiv \{c_t\}_{t=0}^{\infty}$ denotes consumption and β is the discount factor. The instantaneous utility function $u(c_t)$ is assumed to be strictly increasing, strictly concave and continuously differentiable, with

$\lim_{c \rightarrow 0} u'(c) = \infty$ and $\lim_{c \rightarrow \infty} u'(c) = 0$.

Each household is subject to a per-period budget constraint, which is given by

$$c_t + a_{t+1} = w\epsilon_t + (1+r)a_t - \chi,$$

where χ denotes a lump-sum tax and $\epsilon_t > 0$ is an idiosyncratic shock to households' labor productivity, which is assumed to follow a Markov process with transition matrix $\pi(\epsilon_{t+1}|\epsilon_t)$. Households self-insure against income fluctuations by saving in one-period risk-free bonds, denoted by a_{t+1} . Bonds are issued by firms and the government, as in Aiyagari and McGrattan (1998) and Flodén (2001). Bonds issued by firms are claims to physical capital. We abstract from aggregate risk, which implies that claims to physical capital and government bonds are perfect substitutes and thus yield the same return, r .⁴

Households are also subject to the borrowing constraint

$$a_{t+1} \geq \underline{a},$$

where $\underline{a} \leq 0$.

As a key modification with respect to the seminal papers of Aiyagari and McGrattan (1998) and Flodén (2001), households in our economy are allowed to borrow up to a certain limit.⁵

We make the following two additional assumptions regarding the instantaneous utility function and the transition matrix

Assumption 1. $u(c)$ is unbounded below: $\lim_{c \rightarrow 0} u(c) = -\infty$

Assumption 2. $\pi(\epsilon_{t+1}^{min}|\epsilon_t^{min}) > 0$

These assumptions make sure that the natural borrowing limit $\underline{a}^{nat} \equiv -\frac{w\epsilon_{min}-\chi}{r}$, which we define following Aiyagari (1994), will never be binding. This definition of the natural borrowing limit makes sure that households are always able to repay their debt without having to suffer from non-negative consumption, even if they receive the worst possible productivity realization ϵ_{min} forever. By Assumption 2, this event can occur with positive probability. Since by Assumption 1, households' utility approaches $-\infty$ when consumption goes to zero, households will avoid borrowing \underline{a}^{nat} , because this lead to utility of $-\infty$ with positive probability.

Before turning to the government's problem, we state the households' problem in recursive notation:

$$\begin{aligned} W(a, \epsilon; \theta) &= \max_{c, a'} \left\{ u(c) + \beta \sum_{\epsilon'} \pi(\epsilon'|\epsilon) W(a', \epsilon'; \theta) \right\} \\ \text{s.t. } c + a' &= w\epsilon + (1+r)a - \chi \\ a' &\geq \underline{a}, \end{aligned} \tag{2}$$

where θ denotes the distribution of households across a and ϵ , which is a parameter in the households' problem, as it determines aggregate prices r and w . We only analyze stationary environments where θ

⁴In Gomes, Michaelides, and Polkovnichenko (2008) and Gomes, Michaelides, and Polkovnichenko (2010), government bonds and private capital are imperfect substitutes due to aggregate uncertainty.

⁵Borrowing by households can be interpreted as bonds that are issued to other households ('IOUs') or to the government.

and therefore all aggregate variables are constant over time. We will present the exact definition of our stationary equilibrium below. First, we briefly introduce the government's and the firms' problem.

Government. We assume that the government finances interest payments for the stock of government debt B by levying lump-sum taxes χ . Abstracting from other government expenditures, the (stationary) government's budget constraint reads as

$$rB = \chi. \quad (3)$$

Firms. There is a continuum of firms which operate a production technology with constant returns. Firms take prices on all markets as given. We can therefore consider a representative firm which produces output Y using capital K and labor L as inputs:

$$Y = F(K, L),$$

where the technology F is strictly increasing, strictly concave and continuously differentiable in both arguments. We further assume that the Inada conditions are satisfied. The representative firm rents its inputs capital K and labor L at prices r and w , respectively.

Stationary equilibrium. We are now ready to define a stationary equilibrium for our simplified economy:

Definition 1. (*Stationary Equilibrium - Simplified Economy*) Given a transition matrix π and a government policy B , a stationary equilibrium is defined by a distribution of asset and income states $\theta(a, \epsilon)$, factor prices $(r, w) = (r(\theta), w(\theta))$, lump-sum taxes χ , a value function $W = W(a, \epsilon)$ and policy functions $c(a, \epsilon)$, $a'(a, \epsilon)$ such that

1. Households' maximize problem (2).
2. Competitive firms maximize profits: factor prices are given by $r = F_K(K, L) - \delta$ and $w = F_L(K, L)$.
3. The government budget constraint (3) holds.
4. Market clearing: $\int \epsilon d\theta(\epsilon, a) = L$, $A = \int a' d\theta(\epsilon, a) = K + B$ and $\int c d\theta(\epsilon, a) + G + I = F(K, L)$, where $I \equiv \delta K$.
5. Rational expectations: expected law of motion $\theta'(a', \epsilon') = \Gamma[\theta(a, \epsilon)]$ is the true law of motion.
6. Stationarity of θ : $\theta(a', \epsilon') = \Gamma[\theta(a, \epsilon)]$.

We now analyze the role of government debt in providing liquidity to private households. We will show that the effectiveness to which the government can provide liquidity depends on the specification of the borrowing constraint \underline{a} .

2.2 Government Debt and the Effective Borrowing Limit

In order to make the link between public debt and private liquidity more obvious, we follow Aiyagari and McGrattan (1998) and rewrite the households' budget constraint by substituting in the government's budget constraint:

$$\begin{aligned} c + a' &= w\epsilon + (1+r)a - \underbrace{\chi}_{=rB} \iff \\ c + \hat{a}' &= w\epsilon + (1+r)\hat{a}, \end{aligned}$$

where $\hat{a} \equiv a - B$ denotes the assets of private households, net of public debt. We now have an 'effective' borrowing constraint $\hat{a}' \geq \hat{a}$, where $\hat{a} \equiv \underline{a} - B$. This effective borrowing limit depends negatively on public debt B , implying that more public debt leads to a 'looser' effective debt limit. This notation captures the idea that government bonds are claims against the future labor income of households. Government bonds relax the effective borrowing limit by allowing constrained households to consume today and pay in the future in form of taxes, an observation which is due to Woodford (1990). Therefore, government debt provides additional liquidity to private households, because government bonds can be bought in good times and sold in bad times.

By construction, the effective borrowing constraint \hat{a} does not only depend on B , but also on \underline{a} . Therefore, the degree to which government debt can provide liquidity to private households, i.e. relax their effective borrowing limit, will also depend on the impact of government debt on \underline{a} .

In the following, we analyze the relationship between government debt and effective borrowing limits for three different cases:

1. \underline{a} is constant and exogenous with respect to public debt. \underline{a} is sometimes also called an *ad-hoc* debt limit. This is the standard case assumed in the literature that analyzes the impact of public debt in Aiyagari (1994)-type of environments, see, for example, Aiyagari and McGrattan (1998), Flodén (2001), Desbonnet and Weitzblum (2011), Guerrieri and Lorenzoni (2011) or Röhrs and Winter (2014). We assume that $\underline{a} > \underline{a}^{nat}$, in order to ensure that the constraint is binding for some households.
2. The private borrowing limit is given by the natural borrowing limit $\underline{a}^{nat} \equiv -\frac{w\epsilon_{min}-\chi}{r}$. The natural borrowing limit is sometimes labeled as endogenous borrowing limit (see e.g. Eggertson and Krugman 2012).
3. The private borrowing limit is endogenous and determined as an equilibrium outcome, i.e. $\underline{a}(\epsilon; \theta)$. We follow Zhang (1997), Alvarez and Jermann (2000), Kehoe and Levine (2001), Krueger and Perri (2006) and Ábrahám and Cárceles-Poveda (2010) by assuming that households cannot commit to honor their debt contracts. Upon default, households live in autarky forever. The borrowing limit is determined such that there is no default in equilibrium. It will therefore depend on the realization of the individual productivity shock ϵ and the distribution of households θ , which in turn determines aggregate variables, in particular aggregate prices.

Studying changes of government debt within the framework that is outlined in the third case is mainly of interest to us. However, cases one and two turn out to be important in order to understand case three.

Case 1: Exogenous borrowing limit. We show that in this scenario, an increase in government debt translates one-to-one into looser effective private borrowing limits. We also show that raising government debt leads to a higher interest rate in this environment.

The close connection between government debt and effective borrowing limits follows from the definition of $\hat{a}' \geq \hat{a}$ and the fact that \underline{a} is exogenous. Therefore, $\Delta B = -\Delta \hat{a}$. This result has important policy implications. Suppose that \underline{a} tightens, a fact that many scholars have seen as the starting point of the recent financial crisis (see e.g. Hall (2011)). Then, this tightening could be fully offset by the government by adjusting the supply of government bonds accordingly. See e.g. Guerrieri and Lorenzoni (2011) and Eggertson and Krugman (2012).

An increase in public debt B also raises the interest rate, because the supply of assets increases more than their demand, which is due to the fact that the borrowing constraint is binding for some households. These households do not absorb the additional amount of government bonds in order to offset the future increase in taxes (this is exactly how government debt relaxes the effective borrowing constraint in the first place). Put differently, $\Delta A(r) < \Delta B$, where $A(r) \equiv \int a'(r) d\theta(\epsilon, a)$. Since asset market clearing requires $\hat{A}(r) = K(r)$, where $\hat{A}(r) \equiv A(r) - B$, it follows that an increase in public debt B has to crowd out private capital $K(r)$. From the properties of the production function, this results in a higher equilibrium interest rate r . A higher interest rate r in turn facilitates precautionary saving, which is another way to say that an increase in government debt helps to alleviate borrowing constraints, see e.g. Flodén (2001). Laubach (2009) empirically documents for the US that an increase in government debt indeed raises the interest rate.

Case 2: Natural borrowing limit. In this case, a change in public debt has no impact on the effective private debt limit, private capital accumulation or aggregate prices. Recall that $\underline{a}^{nat} \equiv -\frac{w\epsilon_{min}-\chi}{r}$. Since the government's budget constraint implies $\chi = rB$, an increase in public debt B changes \underline{a}^{nat} one-to-one and $\Delta B = \Delta \underline{a}^{nat}$. Therefore, $\Delta \hat{a}^{nat} = 0$, where $\hat{a}^{nat} \equiv \underline{a}^{nat} - B$.

Hence, an increase in B implies that $\Delta A(r) = \Delta B$. It therefore follows that a change in the stationary level of B does not have any impact on $\hat{A}(r) \equiv A(r) - B$ and therefore also neither on $K(r)$ nor on the equilibrium interest rate r itself.

Case 3: Private borrowing limit is endogenously determined as an equilibrium outcome. In order to derive the borrowing limit in this case, we start by specifying the value of autarky:

$$V(\epsilon; \theta) = u((1 - \lambda)w(\theta)\epsilon - \chi) + \beta \sum_{\epsilon'} \pi(\epsilon'|\epsilon) V(\epsilon'; \theta), \quad (4)$$

where we assumed stationarity of the equilibrium distribution: $\theta(a', \epsilon') = \theta'(a', \epsilon') = \Gamma[\theta(a, \epsilon)]$. λ , if positive, denotes an additional penalty for defaulting, as in Livshits, MacGee, and Tertilt (2007). This penalty captures, as a reduced form, different monetary/non-monetary costs of defaulting, such as the fraction of income garnished by lenders, the social stigma of defaulting or the fixed monetary costs of

filing. See Ábrahám and Cárceles-Poveda (2010) for additional examples. λ will become important in the quantitative part in order to match the fraction of households in debt.

For households honoring the financial contract, the optimization problem can be stated as follows:

$$\begin{aligned} W(a, \epsilon; \theta) &= \max_{c, a'} \left\{ u(c) + \beta \sum_{\epsilon'} \pi(\epsilon'|\epsilon) W(a', \epsilon'; \theta) \right\} \\ \text{s.t. } c + a' &= w(\theta)\epsilon + (1 + r(\theta))a - \chi \\ a' &\geq \underline{a}(\epsilon'; \theta) \text{ for all } \epsilon'|\epsilon \text{ with } \pi(\epsilon'|\epsilon) > 0, \end{aligned} \quad (5)$$

where $\underline{a}(\epsilon'; \theta)$ denotes the lower bound on private assets. This lower bound is state-dependent, since there is one for each continuation state that is reached with positive probability in the next period. $\underline{a}(\epsilon'; \theta)$ is the tightest among the state-dependent lower bounds. The state-dependent lower bounds can be seen as default thresholds, such that households who face ϵ and θ are indifferent between defaulting and honoring their debt contracts, see Ábrahám and Cárceles-Poveda (2010). We therefore get an additional equilibrium constraint:

$$\begin{aligned} \underline{a}(\epsilon; \theta) &= \{ \underline{a}(\epsilon; \theta) : W(a = \underline{a}(\epsilon; \theta), \epsilon; \theta) = V(\epsilon; \theta) \} \\ a'(\epsilon'; \theta) \geq \underline{a}(\epsilon; \theta) &\equiv \sup_{\epsilon' : \Pi(\epsilon'|\epsilon) > 0} \{ \underline{a}(\epsilon'; \theta) \}. \end{aligned} \quad (6)$$

Two remarks are in order. First, if $\Pi(\epsilon'|\epsilon) > 0 \forall \epsilon$, i.e. all future shocks will be reached with positive probability, the effective lower bound that households face tomorrow does not depend on the current realization of ϵ . Second, there are many lower bounds that prevent default in equilibrium. Condition (6) states that we are interested in the loosest one, which makes households just indifferent between defaulting and non-defaulting.

One can then show that there is a unique, non-positive and finite default threshold $\underline{a}(\epsilon; \theta)$ such that $\underline{a}^{nat} < \underline{a}(\epsilon; \theta) \leq 0$. All proofs are relegated to Appendix B.

Proposition 1. (*Ábrahám and Cárceles-Poveda 2010*): *Under Assumption 1, condition (6) defines a unique, non-positive and finite default threshold $\underline{a}(\epsilon; \theta)$ for every ϵ and θ*

Corollary 1. *Under Assumptions 1 and 2, the unique default threshold $\underline{a}(\epsilon; \theta)$ is such that $\underline{a}^{nat} < \underline{a}(\epsilon; \theta) \leq 0$. $\underline{a}(\epsilon; \theta)$ is binding for a positive fraction of the population.*

The fact that $\underline{a}(\epsilon; \theta)$ is binding for some households is important, because it implies that an increase in government debt will crowd out private capital and therefore lead to a higher interest rate r . This result establishes a direct link between $\underline{a}(\epsilon; \theta)$ and government debt, as a higher r leads to a tighter $\underline{a}(\epsilon; \theta)$, all other things equal:

Proposition 2. *Suppose that there is an equilibrium distribution $\theta(a, \epsilon)$ that is consistent with a stationary equilibrium as specified by Definition 1. The associated aggregate prices are then given by $r(\theta)$ and $w(\theta)$, and the lump-sum tax that clears the government's budget constraint by $\chi(\theta)$. Moreover, $\underline{a}(\epsilon; r(\theta), w(\theta), \chi(\theta))$ is the associated equilibrium default threshold, which is determined by condition (6).*

Now consider an interest rate $r^+ > r(\theta)$. Condition (6) together with r^+ , $w(\theta)$ and $\chi(\theta)$ imply a (partial) equilibrium default threshold $\underline{a}^+(\epsilon; r^+, w(\theta), \chi(\theta))$, such that $\underline{a}^+(\epsilon; r^+, w(\theta), \chi(\theta)) \geq \underline{a}(\epsilon; r(\theta), w(\theta), \chi(\theta))$ for every ϵ and θ .

The intuition behind this result is straightforward. A higher interest rate raises the debt burden for borrowers, and therefore increases their temptation to default. The default threshold thus needs to be tighter, in order to prevent default in equilibrium.

Two remarks regarding Proposition 2 are in order. First, as long as there is borrowing in equilibrium, i.e. $\underline{a}(\epsilon; \theta) < 0$, then an increase in the interest rate will have a positive effect on the equilibrium default threshold. Second, the response of the equilibrium default threshold to a change in the interest rate does not directly depend on the default penalty λ . Of course, λ has an indirect effect, since it determines the level of the equilibrium default threshold $\underline{a}(\epsilon; \theta)$, and therefore whether $\underline{a}(\epsilon; \theta)$ is zero or negative. In our calibrated model, λ will be set such that $\underline{a}(\epsilon; \theta) < 0$, which is what we observe in the data as well.

Proposition 2 implies that $\Delta \hat{\underline{a}}(\epsilon; \theta) < -\Delta B = \Delta \hat{\underline{a}}$, where $\hat{\underline{a}}(\epsilon; \theta) \equiv \underline{a}(\epsilon; \theta) - B$. We call this an attenuation effect of our equilibrium borrowing limit, because government debt becomes less effective at providing liquidity to private households, relative to an ad-hoc borrowing limit.

Notice, however, that changes in government debt do not only affect $\underline{a}(\epsilon'; \theta)$ through changes in the interest rate r , but also via changes in the wage rate w and in lump-sum tax χ . Both w and χ also affect the equilibrium default threshold, but we do not know their exact direction. The extent to which there is an attenuation or rather an amplification effect needs to be answered with the help of a quantitative model. Corollary 1 only implies that $\hat{\underline{a}}(\epsilon; \theta)$ is bounded such that $\hat{\underline{a}}^{nat} < \hat{\underline{a}}(\epsilon; \theta) \leq -B$, but does not make any statements about $\Delta \hat{\underline{a}}(\epsilon; \theta)$.

As a final remark, note that a change in public debt B that is large enough such that \underline{a}^{nat} approaches $\underline{a}(\epsilon'; \theta)$ (or even 0) from below will always imply that $\Delta \hat{\underline{a}}(\epsilon; \theta) < -\Delta B = \Delta \hat{\underline{a}}$ (or even $0 < \Delta \hat{\underline{a}}(\epsilon; \theta) < -\Delta B$), such that $\hat{\underline{a}}^{nat} < \hat{\underline{a}}(\epsilon; \theta) \leq -B$.

Discussion: Saving after default, temporary exclusion from credit and default in equilibrium. We now discuss the implications of several extensions of our framework on the relationship between private borrowing limits and government debt. All extensions are motivated by the actual bankruptcy procedures observable in the US.

Following Kocherlakota (1996) and others, it became standard to assume that households are completely excluded from financial markets forever after default, i.e. that both borrowing and saving are not possible. In this case, default triggers the worst possible subgame perfect equilibrium, where consumption is equal to income forever. Arguably, while in reality it might be possible to deny future credit to households that have filed for bankruptcy as long as they appear on public records, it might be more difficult to prevent them from accumulating savings. We therefore relax this assumption by imposing that only borrowing is not permitted. The details of this model extension are delegated to Appendix A. Here, we summarize the results and their implications for the trade-off between government debt and private liquidity.

We find that allowing for saving after default will lead to tighter borrowing limits. The precise impact of saving on debt limits depends on the particular specification. In the extreme case where households are allowed to save in all periods, i.e. also in the period when they default on their debt, we

can show that only non-negative borrowing limits prevent default in equilibrium. In other words, there will be no equilibrium borrowing at all, unless the default penalty λ is large enough.

Intuitively, the option to save after default enables households to self-insure against productivity shocks in exactly the same way as they can self-insure if they continue honoring the debt contract. Default becomes the preferred option by debtors, since this allows them to get rid of their debt and the associated debt payments. This finding is reminiscent of the well-known Bulow and Rogoff (1989) result in the context of sovereign debt. Bulow and Rogoff (1989) show that lending to a small country cannot be supported by reputation for repayment if the country has access to insurance contracts.

Although this result hinges on the fact that households can save when defaulting, which is only partially possible according to the US bankruptcy code, we show that permitting for saving after default always leads to tighter debt limits compared to the benchmark case.⁶ In other words, allowing for saving after default only has an effect on the level of the equilibrium borrowing constraint, but not its on the responsiveness to a change in public debt. If anything, the option to save therefore reinforces the positive effect of public debt on the equilibrium borrowing limit, since the interest rate has a positive impact on the value of default if households own wealth. Government debt therefore becomes even less effective at providing liquidity for private households.

For the same reasons, assuming temporary instead of permanent exclusion leads to tighter equilibrium borrowing limits as well, unless the default penalty λ is increased accordingly. Moreover, compared to the benchmark model with permanent exclusion, temporary exclusion increases the sensitivity of the equilibrium borrowing constraint to changes in the interest rate and therefore also to government debt. This is because, by construction, households re-enter the credit market with non-negative assets. As a result, government debt becomes less efficient at relaxing private borrowing limits, relative to the benchmark with permanent exclusion.

Note that so far, we have simply imposed the equilibrium borrowing limits, without further specifying the structure of the financial intermediation sector. Ábrahám and Cárceles-Poveda (2010) show that they could arise as an equilibrium outcome if competitive financial intermediaries were allowed to set them. However, this result depends on the assumption that intermediaries cannot set different interest rates for borrowers and savers. If they could, there would be default in equilibrium, and the interest rate for borrowers would be higher than the one of savers. The papers by Athreya (2002), Chatterjee et al. (2007) and Livshits, MacGee, and Tertilt (2007) model how US households can file for bankruptcy under Chapter 7 of the US bankruptcy code. Athreya (2005) provides an overview over this literature strand. In these papers, households are excluded from borrowing for a certain number of periods. Lenders anticipate the default probabilities that are associated with different combinations of assets and productivity shocks. They charge individual specific borrowing rates, so that they get, on average, the market return, i.e. the interest rate that is paid to savers.

In a model with equilibrium default, the interest rate spreads between borrowing and lending rates, and not borrowing constraints, measure the tightness of the credit market imperfection. In the following, we argue that the impact of changes in government debt on credit market imperfections in a model with limited commitment is independent of the presence of equilibrium default. By assumption, government debt is free of default risk. This means that in a model in which households default in equilibrium, the

⁶Households can keep a maximum amount of assets. The maximum amount varies across US states. See Quadrini and Ríos-Rull (2014).

government would have to pay the market interest rate for its debt, which is lower than the interest rate charged for private debtors. Accordingly, Ricardian equivalence breaks down, and an increase in public debt leads to a higher market interest rate. An increase in the risk-free rate, in turn, raises the average borrowing rate such that the interest rate spread increases, see Livshits, MacGee, and Tertilt (2010). This is because a higher interest rate deters low-risk borrowers, leading to an increase in the risk-premium on average. In our environment instead, a higher interest rate translates into tighter borrowing limits, such that households do not have incentives to default.

To sum up, allowing for saving after default leads to tighter equilibrium debt limits, for a given default penalty λ , independently of whether the exclusion from borrowing is temporary or permanent. Moreover, the option to save introduces a positive link between the interest rate and the default value, therefore increasing the responsiveness of the default value to a change in the interest rate. All other things equal, government debt becomes less effective at providing liquidity to private households, relative to the benchmark case with permanent exclusion and no saving. Hence, we conclude that our model provides a lower bound for the trade-off between public debt and private liquidity. We also expect that an increase in government debt would raise the spread between borrowing and lending rates in a model with equilibrium default. We leave the analysis of the quantitative implications of these extensions for future research.

3 Quantitative Model

We introduce endogenous labor supply, distortive (linear) taxation and exogenous technological progress into the model from the previous section. With the help of this extended model, we then analyze the link between government debt and private borrowing limits quantitatively. The endogenous labor-leisure choice constitutes an important self-insurance device, see Pijoan-Mas (2006). All other things equal, adding a labor-leisure choice will diminish the role of borrowing and saving for self-insurance, and will therefore make autarky relatively more attractive.

The financing of government expenditures through distortive taxation affects households' decisions regarding asset demand and labor supply, which in turn influences aggregate prices. Recall that the response of aggregate prices to changes in government debt is important in order to determine the equilibrium borrowing limit. In the presence of distortive taxation, changes in government debt have an impact on aggregate prices via the crowding-out of private capital and via distorting households decisions.

The presence of technological progress implies that households, on average, become richer over time. Technological progress thus increases the propensity of households to borrow.

In the following, we briefly present those parts of the economy that need to be modified, relative to the simplified economy above.

Households. The new version of the recursive problem of households in autarky can be expressed

as follows:

$$\begin{aligned}
V(\epsilon; \theta) &= \max_{c^{aut}, l^{aut}} \left\{ u(c^{aut}, l^{aut}) + \beta \sum_{\epsilon'} \pi(\epsilon'|\epsilon) V(\epsilon'; \theta) \right\} \\
\text{s.t. } c^{aut} &= (1 - \lambda) \bar{w} \epsilon (1 - l^{aut}) - \chi \\
0 &\leq l^{aut} \leq 1,
\end{aligned} \tag{7}$$

where $\bar{w}_t = (1 - \tau_{l,t}) w_t$ denotes the after-tax wage. In order to have a time-invariant problem, we have to assume that the environment is stationary such that $\theta = \Gamma[\theta]$, where θ is the distribution of households over a and ϵ .

For households who do not default, the optimization problem can be stated as follows:

$$\begin{aligned}
W(a, \epsilon; \theta) &= \max_{c, l, a'} \left\{ u(c, l) + \beta \sum_{\epsilon'} \pi(\epsilon'|\epsilon) W(a', \epsilon'; \theta) \right\} \\
\text{s.t. } c + a' &= \bar{w} \epsilon (1 - l) + (1 + \bar{r}) a - \chi \\
a' &\geq \underline{a}(\epsilon; \theta) \text{ for all } \epsilon'|\epsilon \text{ with } \pi(\epsilon'|\epsilon) > 0 \\
0 &\leq l \leq 1,
\end{aligned} \tag{8}$$

where $\bar{r}_t = (1 - \tau_a(a)) r_t$ denotes the after-tax interest rate. The equilibrium borrowing limit is determined following condition (6).

Firms. Relative to the firms' problem discussed above, we now introduce labor-augmenting technological progress. X_t denotes exogenous labor-augmenting technological progress. This technology is assumed to grow exogenously at a constant rate $X_{t+1} = (1 + g)X_t$ with $g > 0$. For simplicity we normalize initial technology to $X_0 = 1$, such that:

$$X_t = (1 + g)^t.$$

Output in period t is then given by

$$Y_t = F(K_t, X_t L_t).$$

In order to obtain a stationary environment, we detrend all variables by dividing through Y_t . The detrended variables are denoted using a 'tilde', e.g. $\tilde{K}_t \equiv \frac{K_t}{Y_t}$. In Appendix C, we outline the detrended version of households' problems (7) and (8).⁷

Government. The government uses a proportional tax rate on labor income, denoted by τ_l , and on asset income, denoted by τ_a , to finance a lump sum transfer $-\chi_t$ and government consumption G .⁸ Only non-negative asset income is taxed. In other words, there is no proportional subsidy for debt. We therefore define the tax rate on asset income τ_a , as a function of assets a , as follows:

$$\tau_a(a) = \begin{cases} \bar{\tau}_a & \text{if } a \geq 0 \\ 0 & \text{if } a < 0 \end{cases}$$

⁷In Appendix C, we illustrate that technological progress reduces the 'effective' discount factor, therefore implying that in the presence of technological progress, households are less willing to save.

⁸We denote a lump sum tax by χ and a lump-sum transfer by $-\chi$.

Along a balanced growth path equilibrium, aggregate capital K_t , aggregate consumption C_t , government consumption G_t , public debt B_t , aggregate transfers TR_t , the aggregate tax base \bar{A}_t and w_t grow at the same constant rate g , while r and L are constant, where $TR_t = \int \chi_t d\theta(a, \epsilon)$ and $\bar{A}_t = \int_{a \geq 0} a_t d\theta(a, \epsilon)$.

This implies that the detrended versions of these variables, namely \tilde{K} , \tilde{C} , \tilde{G} , \tilde{B} , \tilde{TR} , \tilde{A} and \tilde{w} , are constant. This allows us to drop the time index and write the stationary version of the government's budget constraint as follows:

$$\tilde{G} + (r - g)\tilde{B} + \tilde{TR} = \tau_l \tilde{w} L + \bar{\tau}_a r \tilde{A}. \quad (9)$$

The fact that the detrended variables are constant will also be useful to define a stationary equilibrium.

Stationary equilibrium. Using the characterization of the three sectors we can define the stationary equilibrium for our extended economy:

Definition 2. (*Stationary Equilibrium - Quantitative Model*) Given a transition matrix π and a government policy $\tilde{B}, \tau_a(a), \tau_l$, a stationary equilibrium is defined by a stationary distribution of asset and income states $\theta(\tilde{a}, \epsilon)$, government consumption \tilde{G} , factor prices (r, \tilde{w}) , value functions $W = W(\tilde{a}, \epsilon)$ and $V = V(\epsilon)$ and policy functions $c(\tilde{a}, \epsilon)$, $a'(\tilde{a}, \epsilon)$ and leisure $l(\tilde{a}, \epsilon)$ such that

1. Households' utility maximization problem is defined in equation (11).
2. Competitive firms maximize profits, such that factor prices are given by

$$\begin{aligned} \tilde{w} &= F_L(\tilde{K}, L) \\ r &= F_K(\tilde{K}, L) - \delta. \end{aligned}$$

3. The government budget constraint as defined in equation (9) holds.
4. Factor and goods markets have to clear:

- Labor market clearing:

$$N = \int \epsilon(1 - l) d\theta(\tilde{a}, \epsilon) = L.$$

- Asset market clearing:

$$\tilde{A} = \int \tilde{a}' d\theta(\tilde{a}, \epsilon) = \tilde{K} + \tilde{B}.$$

- Goods market clearing:

$$\int \tilde{c} d\theta(\tilde{a}, \epsilon) + \tilde{G} + \tilde{I} = F(\tilde{K}, L).$$

The investment/output ratio \tilde{I} is given by

$$\tilde{I} \equiv \delta \tilde{K}$$

5. Rational expectations of households about the law of motion of the distribution of shocks and asset holdings, Γ , reflect the true law of motion, as given by

$$\theta'(\tilde{a}', \epsilon') = \Gamma[\theta(\tilde{a}, \epsilon)],$$

where $\theta(\tilde{a}, \epsilon)$ denotes the joint distribution of asset holdings/output and productivity shocks.

6. *Borrowing limits are set such there is no default as given by equation (6).*
7. *The distribution of assets and income states is stationary, such that $\theta(\tilde{a}', \epsilon') = \theta(\tilde{a}, \epsilon)$.*

Before we proceed, we would like to note that Propositions 1 and 2 as well as Corollary 1, which we stated in the context of our simplified economy, still apply in the extended model with endogenous labor supply and distortive taxation. See the Proposition 2.1 in Ábrahám and Cárceles-Poveda (2008), which shows that the equilibrium default threshold is unique, finite and non-positive in a framework containing similar features to our quantitative model. Using this result, statements corresponding to Corollary 1 and Proposition 2 can be readily shown.

We now discuss the welfare criterion that will be used later in order to compare the welfare effects of government debt under different specification of the borrowing constraint.

Welfare measure. As a welfare measure, we compute the aggregate value function:

$$\Omega = \int W(\tilde{a}, \epsilon; \theta) d\theta(\tilde{a}, \epsilon). \quad (10)$$

This criterion can either be interpreted as (i) a Utilitarian social welfare function where every individual has the same weight for the planner, (ii) a steady-state ex ante welfare of an average consumer before realizing income shocks and initial asset holdings or (iii) the probability limit of the utility of an infinitely lived dynasty where households utilities are altruistically linked to each other.

4 Calibration

We calibrate our model such that it is consistent with long run features of the US economy. Our calibration procedure is closely related to Ábrahám and Cárceles-Poveda (2010), and, in particular, to Röhrs and Winter (2014). We will refer to the resulting allocation as our benchmark economy. All parameters are shown in Table 1. In the following, we discuss the rationale behind our parameter choices in greater detail.

Instantaneous utility function and production technology. We assume that preferences can be represented by a constant relative risk aversion utility function:

$$u(c) = \frac{(c^\eta l^{1-\eta})^{1-\mu}}{1-\mu}.$$

Note that the coefficient of relative risk aversion, is equal to $1 - \eta + \eta\mu = 1.3$. This is well in the range (between 1 and 3) commonly chosen in the literature. η denotes the share of consumption in the instantaneous utility function. We calibrate η such that the average share of time worked is 0.3. This results in $\eta = 0.31$. This choice implies an aggregate Frisch elasticity of 1.3.⁹ This is broadly in line with the outcome of other macro models in which the Frisch elasticity of the overall population is considered, but an order of magnitude larger than the Frisch elasticity estimated using micro data from

⁹For our choice of the utility function, the Frisch elasticity is given by $(1 - \mu + \eta\mu)/\mu \cdot (T - h)/h$, where T denotes the time endowment (normalized to 1 in our case) and h denotes the fraction of time spend at work, in our case 0.3.

Table 1: Parameter Choices

Parameter	Value	Target	Data	Model
A. Parameters set exogenously				
Capital's share α	0.3	-	-	-
Growth rate, g	0.02	-	-	-
Public Debt/Output, \tilde{B}	0.67	-	-	-
Labor income tax, τ_l	0.28	-	-	-
Asset income tax, $\bar{\tau}_a$	0.36	-	-	-
Transfers/Output, $\tilde{\chi}$	0.083	-	-	-
Curvature utility function, μ	2	-	-	-
Depreciation rate, δ	0.07	-	-	-
B. Parameters calibrated				
Discount factor, β	0.96	Capital/GDP ratio	3.17	3.17
Weight of consumption in the utility function, η	0.31	Average labor supply	0.3	0.3
Default penalty, λ	-0.83	% of HH with no assets or debt	0.24	0.25
Government spending/Output, \tilde{G}	0.15	GBC clearing	-	-
Vector of income states, s	see text	Wealth and earnings distribution	see Table 2	
Markov transition matrix, Π	see text	Wealth and earnings distribution	see Table 2	

Remarks: see main text for more details on the sources of parameters that are set exogenously and for calibration procedure of remaining parameters. The following abbreviations are used: 'GDP' means 'gross domestic product' (output Y in our model), 'GBC' means 'government budget constraint' and 'HH' means 'households'.

prime age workers.¹⁰

We assume that the aggregate technology is given by a Cobb-Douglas production function:

$$F(K, XL) = K^\alpha (XL)^{1-\alpha}.$$

Initial technology is normalized to $X_0 = 1$, such that $X_t = (1 + g)^t$. We set $g = 0.02$, which implies that our economy grows at a rate of 2 percent per year. The parameter α , which denotes the share of capital in total production, is set to 0.3. This implies a labor share of 0.7. The discount factor β is chosen such that the model reproduces a net wealth/output ratio of 3.1 (cf. Cooley and Prescott (1995) or Ábrahám and Cárceles-Poveda (2010)). Because we want to model the part of wealth that is liquid and used for consumption-smoothing, we define wealth as net financial assets excluding housing and other real estate.¹¹ The resulting β is equal to 0.96. The annual depreciation rate δ is set to 7 percent, which is a common value in the literature (see e.g. Trabandt and Uhlig (2011)).

Taxes and government debt. Following Trabandt and Uhlig (2011), we set the labor income tax rate τ_l to 0.28, the asset income tax rate $\bar{\tau}_a$ to 0.36, lump-sum transfers/GDP $\tilde{\chi}$ to 0.083 and the

¹⁰The debate on whether micro and macro elasticities are consistent is ongoing. See Keane and Rogerson (2011) for a summary.

¹¹There is an ongoing debate to what extent housing wealth is used for consumption smoothing. See Fernández-Villaverde and Krueger (2011) and Kaplan and Violante (2010). Heathcote, Storesletten, and Violante (2009) summarize the debate and provide additional references. We will get back to this point below.

debt/GDP ratio \tilde{B} to 0.67 in the benchmark.¹² Government spending/GDP \tilde{G} is set such that the government's budget constraint clears, given all other parameters.

Income process. We now turn to the parametrization of the income process. We need to find a vector of income states s , containing the productivity shocks $\epsilon > 0$, and a transition matrix Π , containing the transition probabilities between the shocks in different periods.

This can be done in several ways. For example, by estimating an AR(1) process using micro-level panel data of individual earnings, such as the Panel Study of Income Dynamics (PSID). The vector of productivity shocks s and the transition matrix Π are then given by a Markov chain approximation of the estimated AR(1) process. A typical finding in this literature is that individual earnings are very persistent. See Abowd and Card (1989) for an early result, and more recently Storesletten, Telmer, and Yaron (2004) and Krueger and Ludwig (2013).

However, as shown by Quadrini and Ríos-Rull (2014), parameterizations of the earnings process that are based on the PSID generate too little wealth inequality in standard Aiyagari (1994)-type incomplete markets models such as ours. This is unfortunate for our purpose, since disciplining the model such that it is consistent with the skewed distribution of earnings and assets observable in the US economy is key for assessing the interaction between government debt and private credit, given that a large fraction of households (24 percent) in the data has no assets or is in debt.

We thus follow a different approach, which is due to Castañeda, Díaz-Giménez, and Ríos-Rull (2003). Castañeda, Díaz-Giménez, and Ríos-Rull (2003) argue that the reason for the failure of PSID estimates to generate enough wealth concentration is the fact that the PSID misrepresents the distribution of earnings, since it does not include the very rich households. Instead of using the PSID, Castañeda, Díaz-Giménez, and Ríos-Rull (2003) propose to focus on an earnings process that is consistent with the cross-sectional dispersion of earnings measured by the Survey of Consumer Finances (SCF). Differences in the sampling design make the SCF better suited to study wealth and income at the top of the distribution, see e.g. Juster, Smith, and Stafford (1999) for a comparison of PSID and SCF. The persistence of the earnings process is then engineered such that it replicates the observable degree of wealth inequality. Hence, the vector of income states and the transition matrix are chosen such that the distribution of earnings and wealth generated by the model are consistent with the data.

Before we describe the calibration of the earning process in greater detail, we turn to our definition of wealth and earnings. Since this paper is about unsecured debt and we do not model housing, we focus on net financial assets. This distinguishes our paper from Castañeda, Díaz-Giménez, and Ríos-Rull (2003), Krusell and Smith (1998) and others, who target the distribution of net worth, defined as the difference between total assets and total liabilities. We follow Ábrahám and Cárceles-Poveda (2010) and calculate net financial assets by excluding the value of residential property, vehicles and direct business ownership from total assets, and the value of secured debt due to mortgages and vehicle loans from total liabilities.¹³ We argue that this measure gives a better representation of the amount of liquid assets that households can use to smooth out income shocks, since residential property and vehicles are not only used as investment goods, but also serve as durable consumption. Moreover, their adjustment is often

¹²Similar values are also reported by Mendoza, Razin, and Tesar (1994).

¹³In terms of SCF variables, we calculate net financial assets as follows: Net Financial Assets = FIN - DEBT + MRTHL + RESDBT + VEHINST.

associated with significant transaction costs, see Fernández-Villaverde and Krueger (2011). Kaplan and Violante (2014) find that a significant fraction of those households that possess a large amount of net worth do not liquidate their assets due to transaction costs in response to temporary shocks and rather live 'hand-to-mouth'.¹⁴

Earnings are defined as labor earnings (wages and salaries) plus a fraction of business income before taxes, excluding government transfers. This definition corresponds to the concept of earnings that is implied by our model.

The distribution of earnings and net financial assets is computed from the 2007 SCF.¹⁵ Table 2 shows that both earnings and net financial assets are very unequally distributed in the data. The richest 20 percent of the population hold more than 90 percent of all financial assets, net of debt. The distribution of earnings is less skewed. Households in the top quintile earn around 60 percent of the total earnings.

We are now ready to discuss the details of the calibration procedure. We choose the vector s of productivity realizations ϵ and the corresponding transition matrix Π such that inequality of earnings and net financial assets generated by the model matches the distributions of earnings and net financial assets from the 2007 SCF as close as possible. For each unknown parameter, we pick one moment from the empirical Lorenz curve of either earnings or net financial assets. Since we work with four productivity states and normalize their mean to 1, this gives us three unknown parameters. The corresponding transition matrix Π then has 16 entries. Since probabilities in each row sum up to 1, this gives us 12 unknown parameters in addition.

We target the five quintiles and the Gini for both the earnings distribution and the net financial asset distribution, leading to 12 targets. Since the distribution of net financial assets is very unequal even within the upper quintile, we additionally target the top 10%, 5% and 1% percentile. This yields 15 targets in total. The 15 unknown parameters are jointly chosen to match the 15 empirical moments.

As a result, we find the following vector of income states:

$$s = \{0.055, 0.551, 1.195, 7.351\}.$$

It should be noted that the highest income state is more than 130 times as high as the lowest income state.

Furthermore, we get the following transition matrix for the income states:

$$\Pi = \begin{bmatrix} 0.940 & 0.040 & 0.020 & 0.000 \\ 0.034 & 0.816 & 0.150 & 0.000 \\ 0.001 & 0.080 & 0.908 & 0.012 \\ 0.100 & 0.015 & 0.060 & 0.825 \end{bmatrix}.$$

As can be seen from the transition matrix, there is a 10 percent probability of moving from the highest income state today to the lowest income state tomorrow. Since moving to the lowest productivity state is associated with a large drop in income, this generates a strong saving motive for income-rich households. The fact that income-rich households have both the opportunity and the motive to accumulate large asset holdings generates the extremely unequal distribution of net financial assets in the model.

¹⁴Also see the references mentioned in Footnote 11.

¹⁵The SCF does not specify the exact fraction of total business income that is attributable to labor and to capital. We define business income from sole proprietorship or a farm as labor earnings, whereas business income from other businesses or investments, net rent, trusts, or royalties is defined as capital income.

Table 2: Distribution of Net Financial Assets and Earnings, 2007 SCF and Benchmark Stationary Equilibrium

	Q1	Q2	Q3	Q4	Q5	upper 10%	upper 5%	upper 1%	Gini
A. Net financial assets									
Data	-1.60%	0.10%	1.64%	8.29%	91.57%	79.64%	66.83%	39.09%	0.90
Benchmark Calibration	-1.57%	0.88%	3.92%	7.23%	89.54%	70.58%	47.03%	13.53%	0.83
B. Earnings									
Data	-0.40%	3.19%	12.49%	23.33%	61.39%	-	-	-	0.62
Benchmark Calibration	0.00%	2.38%	12.58%	22.73%	62.31%	-	-	-	0.65

Remarks: Quintiles (Q1-Q5) denote net financial assets (resp. earnings) of a group in percent of total net financial assets (resp. earnings), similarly for earnings. For the wealth distribution in addition to the quintiles the table also shows the percent of net financial assets held by the wealthiest 10% (upper 10%), 5% (upper 5%) and 1% (upper 1%). The entries in 'data' are computed from the 2007 SCF. See main text for precise definitions. Notice that earnings can be negative due to the fact that labor earnings also contain part of the gains (or losses) of small enterprises.

The same mechanism is also present in the transition matrix found by Castañeda, Díaz-Giménez, and Ríos-Rull (2003). As noted by Quadrini and Ríos-Rull (2014), the combination of particular values is somewhat arbitrary. But they are informative about how extreme both saving motives and opportunities have to be in the US economy in order to rationalize the skewed wealth distribution in a model in which inequality arises from self-insurance against temporary productivity shocks only.

Three remarks regarding our calibration procedure are in order. First, inequality might be driven by other mechanism than self-insurance, for example entrepreneurship (as in Quadrini 1999 or Cagetti and De Nardi 2006) or discount factor heterogeneity, as in Krusell and Smith (1998). Other important factors that we abstract from are life-cycle considerations and bequests, see e.g. Huggett (1996) or Nardi (2004). Sabelhaus and Song (2010) even find evidence for age-specific earnings processes. However, Gourinchas and Parker (2002) argue that a large fraction of non-pension wealth (up to 70 percent) in the US can be attributed to precautionary savings. Only the residual fraction is due to life-cycle motives.¹⁶

Second, we note that due to the extremely skewed distribution of net financial assets, matching the top percentiles is very difficult. We therefore assign less weight to these targets. As a result, our calibration matches the quintiles of both the distribution of earnings and net financial assets as well as their respective Gini quite well. It fails, however, to generate sufficient asset holdings among the top 5%, and in particular among the top 1% of the net financial asset distribution. Results reported by Quadrini and Ríos-Rull (2014) (see their Table 6) for other calibrations that also use the Lorenz curve of wealth as a target indicate that it is indeed hard to match the extreme right tail of the distribution.

Finally, we would like to emphasize that the high persistence implied by our calibration for the transition matrix Π , in particular for low productivity states, is in line with the empirical evidence on earnings persistence using PSID data mentioned earlier. The high persistence is important for our quantitative results, as the next section will make clear. In short, due to the high persistence of productivity shocks for low-income households, whose behavior is relevant for determining the borrowing limit, a change in the wage rate affects the autarky value and the value of obeying the contract in roughly

¹⁶As a robustness check, we repeated our quantitative exercise using the Krusell and Smith (1998) approach of generating inequality. Preliminary results, which are available from the authors upon request, suggest that our findings are qualitatively robust to using this very different approach of generating inequality.

the same way. This is because low productivity households have to work hard regardless of whether they are in autarky or not. As a consequence, only changes in the interest rate have an impact on the equilibrium borrowing constraint, implying that borrowing constraints become tighter, the more debt is issued, as predicted by our analytical framework.

Before we explain our quantitative results in greater detail, we turn to the calibration of the default penalty λ .

Borrowing limit. We calibrate the default penalty λ to set the equilibrium borrowing limit $\tilde{a}(\epsilon, \theta)$ such that our model is consistent with the percentage of households with negative or zero financial assets in the 2007 SCF. This results in $\lambda = -0.83$.¹⁷ The implied borrowing limit is $\tilde{a}(\epsilon, \theta) = -0.3$.

5 The Trade-Off Between Public and Private Provision of Liquidity in a Quantitative Model

We now use our quantitative framework to analyze the link between public debt and private borrowing limits. We find that if borrowing limits are determined endogenously as an equilibrium outcome, an increase in debt/GDP leads to tighter borrowing constraints. Independently of the specification of the borrowing limit, we illustrate that higher debt/GDP ratios relax the effective borrowing constraint, i.e. make the borrowing limit less binding. Important differences in the strength of this relaxation effect emerge between the ad-hoc limit and the equilibrium borrowing constraint. Public debt is less powerful at relaxing the effective borrowing when the constraint responds endogenously. Our quantitative results therefore confirm our theoretical predictions from Section 2.1.

Technically, we proceed as follows. We use the calibrated parameters of our benchmark economy. As an ad-hoc limit, we set $\underline{a} = -0.3$, which corresponds to the equilibrium default threshold at the benchmark debt/GDP ratio of 0.67.

It is important to note that we analyze different stationary equilibria, without considering the transition between them. This is in line with the seminal work by Aiyagari and McGrattan (1998), Flodén (2001) and others. Computing the transition in our model with equilibrium borrowing constraints is quite involved, since borrowing constraints change in every period along the transition path, in addition to the aggregate prices w and r . Therefore, to determine the equilibria over the transition path, an additional fixed point problem would have to be solved.¹⁸ We discuss potential implications of incorporating the transition when we present our results below.

When computing different stationary equilibria, we vary the public debt/GDP ratio, keeping government expenditures/GDP, \tilde{G} , and transfers/GDP, \tilde{TR} , constant at their benchmark level. This means

¹⁷This implies that there λ is a default benefit rather than a default penalty. The reason for this outcome can be found in the transition matrix Π , which implies that $\Pi(\epsilon^{min}|\epsilon) > 0 \forall \epsilon$. Since it turns out that $\sup_{\epsilon': \Pi(\epsilon'|\epsilon) > 0} \{\underline{a}(\epsilon'; \theta)\}$ is always achieved for $\epsilon' = \epsilon^{min}$, this implies that the effective lower bound on assets tomorrow does not depend on the current realization of ϵ . Hence, all households face the same borrowing limit, which is set such that households with the lowest possible productivity realization ϵ^{min} are indifferent between defaulting and repaying their debt. Since Π implies a high degree of persistence, the autarky utility associated with ϵ^{min} is quite small. This would imply a very tight borrowing limit and therefore only a small fraction of households in debt. Thus, in order to make sure that our model matches the large fraction of households with zero or negative assets, we need a λ of -0.83 .

¹⁸See Ábrahám and Cárceles-Poveda (2010) for an application with exogenous labor supply.

that taxes have to adjust to keep the government's budget constraint balanced. We adjust either the labor income tax rate τ_l or the asset income tax rate $\bar{\tau}_a$. In the main text, we focus on the labor income tax and relegate the results for the asset income tax to Appendix D.

We analyze stationary equilibria that are associated with public debt/GDP ratios between -1 and 2 . A negative debt/GDP ratio means that the government holds assets. These assets are used by firms as capital in the production process or they are lent to private households in form of loans. In either case, the government receives income, which is used to finance government expenditures and transfers. Government expenditures and transfers always exceed the government's asset income for public debt/GDP greater or equal than -1 , so that the tax rates remain positive for all debt/GDP ratios that we analyze.

In reality, most countries, perhaps with the exception of Norway, have positive debt/GDP ratios. We nevertheless included negative debt/GDP ratios in our analysis because, as we will show later, the debt/GDP ratio that maximizes welfare across stationary equilibria is negative. Restricting our analysis to positive debt/GDP ratios would mean that we end up with a corner solution.

The trade-off between public debt and private credit. Equilibrium borrowing limits become more restrictive if the government increases the amount of public debt, relative to GDP. This can be seen from Figure 1 (first panel), where we plot the borrowing limits for various stationary equilibria, which are characterized by different debt/GDP ratios. Hence, public debt crowds out the supply of private credit. Notice that we plot the borrowing limit for households that are subject to the lowest income shock only, as this is the limit applied to all households.¹⁹

In order to construct Figure 1, we adjust the labor income tax such that the government's budget constraint is satisfied for different public debt/GDP ratios. The results for the case in which the asset income tax is adjusted are similar, and are therefore delegated to Appendix C.

As we documented in Section 2.1, the response of the interest rate and the wage rate to changes in public debt are important drivers of the equilibrium debt limit. In the second panel of Figure 1, we plot the interest rate and the wage rate for different stationary equilibria that vary according to their debt/GDP ratio. Perhaps not surprisingly, higher debt/GDP ratios are associated with a higher interest rate and a lower wage rate. Since Corollary 1 implies that the equilibrium borrowing constraint binds, raising debt/GDP crowds-out private capital, therefore causing the observable price changes.²⁰ In particular, an increase in the stationary debt/GDP ratio by one percentage points raises the interest rate by about 1.25 to 1.5 basis points. Laubach (2009) documents that an increase in the (projected) debt/GDP ratio by one percentage point raises the real interest rate in the US by 3 to 4 basis points. The interest rate elasticity implied by our model is therefore smaller than the empirical one. Since, all other things equal, a stronger reaction of the interest rate also leads to a stronger response of the equilibrium borrowing constraint to a change in the debt/GDP ratio (this follows from Proposition 2), we note that our findings may understate the response of the borrowing limit that can be expected from the observable interest rate elasticity.

¹⁹This is due to the specification of the earnings transition matrix, which assigns positive probability of receiving the worst productivity shock in the following period, independently of the current productivity shock.

²⁰Proposition 1 and Corollary 1 can be directly applied to a model with endogenous labor supply, see Ábrahám and Cárceles-Poveda (2008), in particular Proposition 2.1. therein.

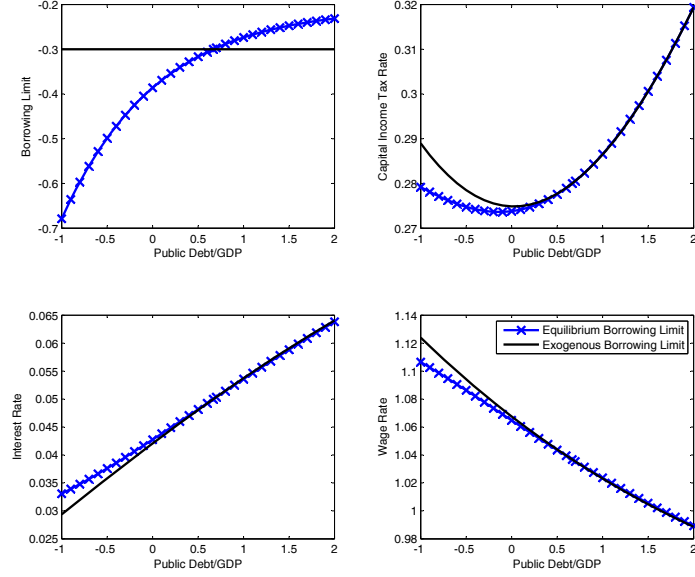


Figure 1: Implied Borrowing Limits, Tax Rates, Interest Rates and Wage Rates for Different Stationary Equilibria. In this exercise we plot the changes in the borrowing limits (first panel), tax rates (second panel), interest rates (third panel) and wage rates (fourth panel) implied by our model for different stationary equilibria that differ with respect to the public debt/GDP ratio. In the benchmark public debt amounts to 2/3 of GDP. The labor income tax is adjusted to balance the budget. Two cases: (i) exogenous (fixed) borrowing limit (black line); (ii) equilibrium borrowing limit (blue line with crosses).

Note that the development of the interest rate and wage rate work in different directions. While the increase in the interest rate raises the temptation to default, which in turn makes lenders more reluctant to give credit, the decline in the wage rate makes the autarky option less attractive for households, all other things equal. In autarky, households are excluded from financial markets, which means that they can rely only on adjustments in their labor supply in order to smooth their consumption across periods. Hence, a lower wage rate should increase the willingness of private lenders to provide credit in stationary equilibria with higher debt/GDP ratios.

It turns out that in our model, the increase in the interest rate and the associated rise in the relative value of defaulting dominates the effect of a decrease in the wage rate. This is because households in the low-income group, whose behavior is relevant for determining the borrowing limit, have to work hard regardless of whether they are in autarky or not. Since our calibration implies that income shocks are very persistent, the change in the wage rate affects the autarky value and the value from obeying the contract in roughly the same way. In contrast, the change in the interest rate only affects the value of debt repayments.

In sum, our results show that increasing the stationary public debt/GDP ratio leads to tighter equilibrium constraints. To what extent this result also holds when the transition between two stationary equilibria is incorporated depends on, among other things, the path of aggregate variables and prices. If they converge monotonically to the new stationary equilibrium, we would expect that also the equilibrium borrowing constraint adjusts monotonically to its new level. Another important determinant of the dynamics of the equilibrium borrowing limit is the behavior of taxes over the transition. If taxes

adjusts such that the autarky option becomes relatively unattractive during the transition, this might lead to reversals in the adjustment of the equilibrium borrowing limit. The interaction between tax changes and equilibrium borrowing constraints over the transition is left for future research.²¹

The impact of public debt on the effective private borrowing limit. We now turn to the question to what extent changes in public debt affect the bindingness of the borrowing constraint. Put differently, we are interested in the relationship between public debt and the effective borrowing constraint. Recall from Section 2.1 that if the government finances its expenditures solely with the help of lump-sum taxation, the effective borrowing constraint is given by $\hat{a}(\epsilon; \theta) \equiv \underline{a}(\epsilon; \theta) - B$.

This definition of the effective borrowing limit can no longer be applied in the context of our quantitative model, since the government levies a linear tax rate on both asset and labor income. In order to measure to what extent borrowing limits are binding, we make use of the fact that according to Corollary 1, $\underline{a}^{nat} < \underline{a}(\epsilon; \theta) \leq 0$. Moreover, Assumptions 1 and 2 imply that the natural borrowing limit \underline{a}^{nat} is the tightest borrowing limit which is not binding. This suggests that we can use the absolute distance between \underline{a}^{nat} and the equilibrium borrowing limit $\underline{a}(\epsilon; \theta)$, defined by $\Delta_{\underline{a}} \equiv |\underline{a}_{nat} - \underline{a}|$, as a measure for the bindingness of $\underline{a}(\epsilon; \theta)$. Intuitively, if $\Delta_{\underline{a}}$ is close to zero, then \underline{a} is not very restrictive. If, however, $\Delta_{\underline{a}}$ is close to \underline{a}_{nat} , this implies, \underline{a} is close to zero, and borrowing limits are relatively tight.

It is also important to note that compared to our simplified framework, we now have to employ a different definition of the natural borrowing limit, namely $\underline{a}^{nat} \equiv -\frac{(1-\tau_w)\tilde{w}\epsilon_{min}-\tilde{\chi}}{r-g}$. Also note that \underline{a}^{nat} depends on tax rates (which in turn also affect aggregate prices) and is therefore not invariant with respect to public debt anymore.

Moreover, different specifications of the borrowing limit lead to different values of the natural borrowing limit. This can be seen in Figure 2, where we plot the ad-hoc borrowing limit of -0.3 , the equilibrium borrowing limit, the natural borrowing limit as well as the distance between them. Figure 2 shows that the distance is falling for higher debt/GDP ratios, independently of the specification of the borrowing limit. Put differently, public debt relaxes effective borrowing limits. Importantly, however, there is an attenuation effect if the borrowing limit arises as an equilibrium outcome, since the impact of debt/GDP on the distance $\Delta_{\underline{a}}$ is much smaller, relative to the case where the borrowing limit is imposed ad-hoc.

This attenuation effect can also be observed in Figure 3, where we show the fraction of households at the borrowing constraint. This fraction declines much more slowly if we generate the borrowing constraint as an equilibrium outcome. In this case, the fraction of constrained households falls by 1.7 basis points if the stationary debt/GDP ratio is increased by one percentage point. If we instead consider the ad-hoc borrowing limit, the decline amounts to 4 basis points, which is more than twice as much.

In sum, our quantitative model shows that an increase in public debt crowds out the provision of private credit if the borrowing limit emerges endogenously as an equilibrium outcome. As a consequence, public debt becomes less powerful in relaxing private borrowing constraints (i.e. in providing liquidity to private households).

In essence, our results suggests that ad-hoc borrowing constraints, which are the dominant choice in the literature, may lead researchers to overstate the liquidity role of government debt. This conclusion

²¹Ábrahám and Cárceles-Poveda (2008) study tax reforms in an environment similar to ours. However, they do not characterize the transition with equilibrium borrowing limits.

is important in light of the recent debate about how to ameliorate the consequences of the financial crisis. A prominent proposal is to increase government debt in order to relax borrowing constraints of private agents, which presumably became tighter during the crisis. See e.g. Guerrieri and Lorenzoni (2011) and Eggertson and Krugman (2012). Our results imply that public debt may be less effective at relaxing debt constraints of private households if constraints respond to public debt.

In their Section 8A, Eggertson and Krugman (2012) also acknowledge that endogenizing the borrowing constraint weakens the impact of fiscal policy on aggregate demand. However, they generate their endogenous borrowing limit as a convex combination of an ad-hoc limit and the natural borrowing constraints. The advantage of our framework is that the borrowing constraint emerges as an equilibrium outcome of a limited commitment problem, which allows us to be explicit about the underlying decision problems that determine the constraint and their response to policy changes. It turns out that modeling the microfoundation of the borrowing constraint is important, since government debt has a very different impact on the equilibrium borrowing limit, compared to the natural borrowing limit and an ad-hoc constraint.

As we will show in the following paragraph, our equilibrium debt limit also substantially alters the welfare implications of government debt.

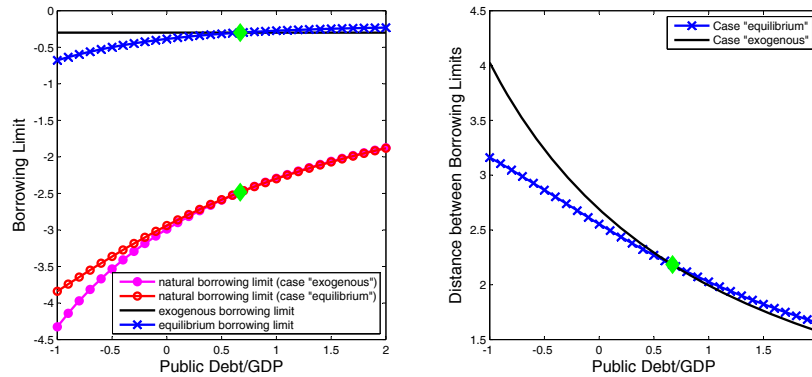


Figure 2: Natural Borrowing Limit, Endogenous Borrowing Limit and Distance Between them. In the left-hand side panel we plot the changes in the natural borrowing limit case 'exogenous' (pink filled circles), the natural borrowing limit case 'equilibrium' (red empty circles) the exogenous (fixed) borrowing limit (black line) and the equilibrium borrowing limit (blue crosses) arising from limited commitment for stationary equilibria differing with respect to the public debt/GDP ratio. In the right-hand side panel we plot the distance between the two borrowing limits for the same set of stationary equilibria. At the benchmark public debt amounts to 2/3 of GDP (green diamond).

Equilibrium borrowing constraints and the welfare implications of government debt.

We now argue that incorporating equilibrium borrowing constraints substantially alters the welfare consequences of changes in public debt. We follow the seminal papers by Aiyagari and McGrattan (1998) and Flodén (2001) and focus on the long-term consequences of changes in the debt/GDP ratio. We do this by comparing Utilitarian welfare of different stationary equilibria. Our contribution is to show that the debt/GDP ratio that maximizes welfare in stationary equilibrium is sensitive to the specification of the borrowing constraints. In particular, we will show that allowing for equilibrium borrowing constraints attenuates the welfare effect of changes in the stationary debt/GDP ratio.

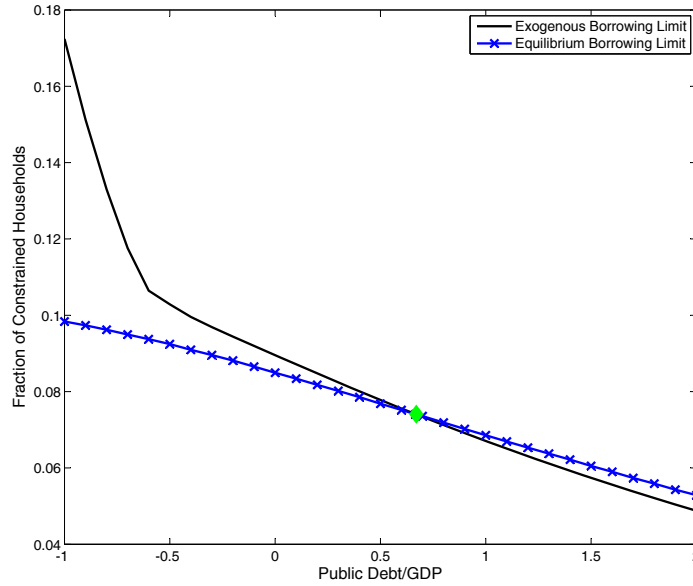


Figure 3: **Fraction of Constrained Households.** Here we plot the number of households at the borrowing constraint for stationary equilibria differing with respect to the public debt/GDP ratio. At the benchmark public debt amounts to 2/3 of GDP (green diamond). Two cases: (i) exogenous borrowing limit (black line); (ii) equilibrium borrowing limit (blue line with crosses).

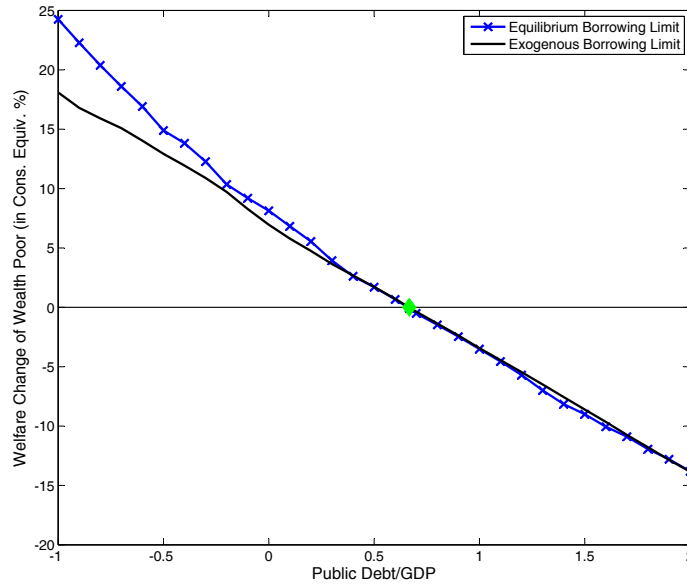


Figure 4: **Welfare of Wealth Poor.** Here we plot the change in consumption equivalent welfare of wealth poor agents holding zero or negative assets, for stationary equilibria differing with respect to the public debt/GDP ratio. At the benchmark public debt amounts to 2/3 of GDP (green diamond). Two cases: (i) exogenous borrowing limit (black line); (ii) equilibrium borrowing limit (blue line with crosses).

We first present the welfare effects for the wealth-poor, i.e. those households with zero or negative assets. Figure 4 reveals that as we move to stationary equilibria associated with higher debt/GDP

ratios, welfare of the wealth-poor drops, relative to the benchmark debt/GDP ratio of 0.67. Strikingly, the decline is more pronounced for the case of the equilibrium constraint.

This is because the equilibrium constraint becomes tighter as we increase debt/GDP. It confirms the findings of Obiols-Homs (2011), who argues that welfare of the wealth-poor is strongly affected by changes in the borrowing constraint.

The fact that welfare declines at all for this group is perhaps puzzling. After all, government debt relaxes the effective constraint, implying that there is an insurance effect of government debt. The reason for the decline is that this positive insurance effect of government debt is more than offset by a negative income composition effect. The latter stems from the fact that a higher debt/GDP ratio depresses the wage rate and therefore also the income of the wealth-poor, which consists of labor income only.

As we mentioned above, we restrict our analysis to a comparison of stationary equilibria. One concern is that by comparing stationary equilibria only, our results might be partly driven by changes in the stationary distribution. Put differently, the 'average' household, for which we compute welfare, is not the invariant to changes in the stationary debt/GDP ratio. By itself, this might already account for part of the aggregate welfare effects, all other things equal.

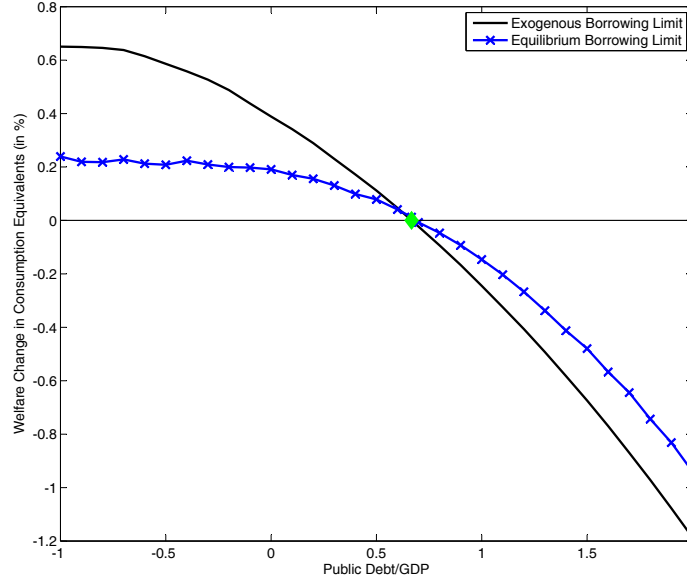


Figure 5: Welfare of Total Population. We plot the welfare change in consumption equivalent units (on the y-axis) for stationary equilibria differing in the public debt/GDP ratio (on the x-axis) for the total population. At the benchmark public debt amounts to 2/3 of GDP (green diamond). Two cases: (i) exogenous borrowing limit (black line);(ii) equilibrium borrowing limit (blue line with crosses).

We can partly address this point by comparing the welfare effects of the wealth-poor (see Figure 4) to those of the overall population (see Figure 5). Under the assumption that the distribution of wealth-poor households is roughly stable across different stationary equilibria, one can infer to what extent aggregate welfare effects are determined by changes in the fraction of wealth-poor households.

The fact that the welfare functions for the wealth-poor and the aggregate population both have a negative slope indicates that the welfare effects must be qualitatively the same. This link can be

explained as follows. The wealth-poor are heavily dependent on labor income in order to finance their consumption. Their welfare therefore depends on the development of the wage rate, which is decreasing with public debt/GDP (recall Figure 1). Because the wealth-poor are also consumption-poor, their marginal utility from consumption is high. Since we evaluate aggregate welfare according to the Utilitarian welfare criterion, changes in the welfare of the wealth-poor have a strong impact on aggregate welfare, as the wealth-poor are also consumption-poor, implying that their marginal utility is high.

But a comparison of Figures 4 and 5 also reveals that the welfare function of the wealth-poor is much steeper than the welfare function for the overall population. This difference in the slope is due to changes in the composition of the population. The lower the debt/GDP ratio, the higher the fraction of wealth-poor households, because low debt/GDP ratios are associated with a low equilibrium interest rate. A low interest rate in turn provides little incentives to save, and the stationary wealth distribution shifts to the left. All other things equal, the higher the fraction of poor households, the lower aggregate welfare. Therefore, despite the fact that a reduction in the stationary debt/GDP ratio is highly beneficial for an individual wealth-poor household, the aggregate welfare gain is much smaller.

Figure 5 also reveals that the aggregate welfare function is flatter for equilibrium borrowing constraints. Hence, incorporating equilibrium borrowing constraints does not only attenuate the response of aggregate prices to changes in public debt/GDP, but also dampens the aggregate welfare consequences of government debt.

We now analyze why the welfare effects of debt/GDP changes are less pronounced in the presence of equilibrium borrowing constraints. We highlight two potential mechanisms. As shown above, under equilibrium borrowing constraints, aggregate prices do not respond as much as under ad-hoc borrowing limits. In particular, we found that changes in public debt have a weaker impact on the wage rate, which in turn is an important determinant of the total income for many households in our economy, given the large fraction of wealth-poor households. The second mechanism that we consider is that the distribution changes differently across stationary equilibria if we compare ad-hoc and equilibrium borrowing limits.

This is indeed the case, as indicated by Figure 6. The fraction of wealth-poor households is much more responsive to changes in public debt/GDP, if the borrowing constraint is generated as an equilibrium outcome.

It is informative to understand how the fraction of wealth-poor households depends on the specification of the borrowing limit. For debt/GDP ratios that are lower than the benchmark, the equilibrium borrowing limit is looser than the ad-hoc limit. This facilitates self-insurance, reduces precautionary savings and therefore increases the fraction of wealth-poor households, which in turn leads to lower aggregate welfare, all other things equal. Instead, for debt/GDP ratios above the benchmark, the equilibrium borrowing constraint is tighter than the ad-hoc limit, there is more need for precautionary savings, and consequently fewer households turn out to be wealth-poor.

Since wealth-poor households have lower welfare, and the fraction of poor households depends on the specification of the borrowing limit, this link could potentially account for the attenuation effect that we observe at the aggregate level.

In order to test whether this is indeed the case, we conduct the following experiment. For all

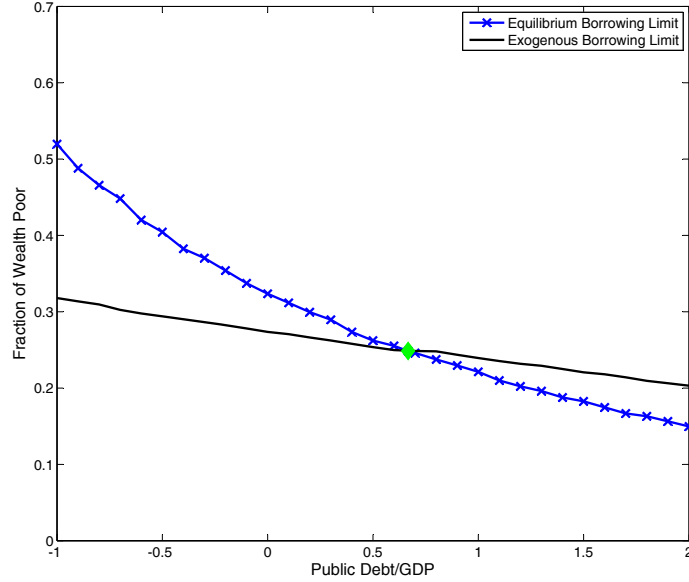


Figure 6: **Fraction of Wealth Poor Households.** Here we plot the fraction of wealth poor households (as a fraction of the total population) implied by our model for different stationary equilibria that differ with respect to the public debt/GDP ratio. In the benchmark public debt amounts to 2/3 of GDP. The labor income tax is adjusted to balance the budget. Two cases: (i) exogenous (fixed) borrowing limit (black line); (ii) equilibrium borrowing limit (blue line with crosses).

debt/GDP ratios, we evaluate welfare at the benchmark distribution. The welfare effects are then solely due to differences in the value function associated with different debt/GDP ratios.

Interestingly, we find that the attenuation effect of equilibrium borrowing constraints is still present, as indicated by Figure 7, where we plot the welfare function for the specification in which the distribution is fixed. For a better comparison, we have also included the welfare function for the case in which the distribution adjusts. The attenuation effect is slightly weaker in the case in which the distribution is kept constant, relative to the overall magnitude of the welfare effects. At the same time, we also observe that the welfare effects become much steeper if we fix the distribution. This finding is due to the fact that aggregate price changes now translate one-to-one into welfare, without being affected by distributional changes.

In sum, our results show that incorporating equilibrium borrowing constraints does not only dampen the impact of changes in the stationary public debt/GDP ratio on aggregate prices, but also on welfare. We find that the attenuation effect of equilibrium constraints on aggregate prices can also explain why equilibrium constraints dampen the impact of changes in debt/GDP on aggregate welfare. Changes in the distribution weaken the impact of aggregate prices on overall welfare.

Before we conclude, we would like to discuss several other caveats that arise because we do not consider the transition. Recent papers using models with exogenous borrowing constraints, in particular Desbonnet and Weitzblum (2011) and Röhrs and Winter (2014), have shown that transitional welfare effects are quantitatively important and can outweigh the long-term effects. This short-run effects arise because of changes in the tax rates that are necessary to keep the government's budget constraint balanced. For example, a reduction in government debt needs to be financed by increasing taxes, which

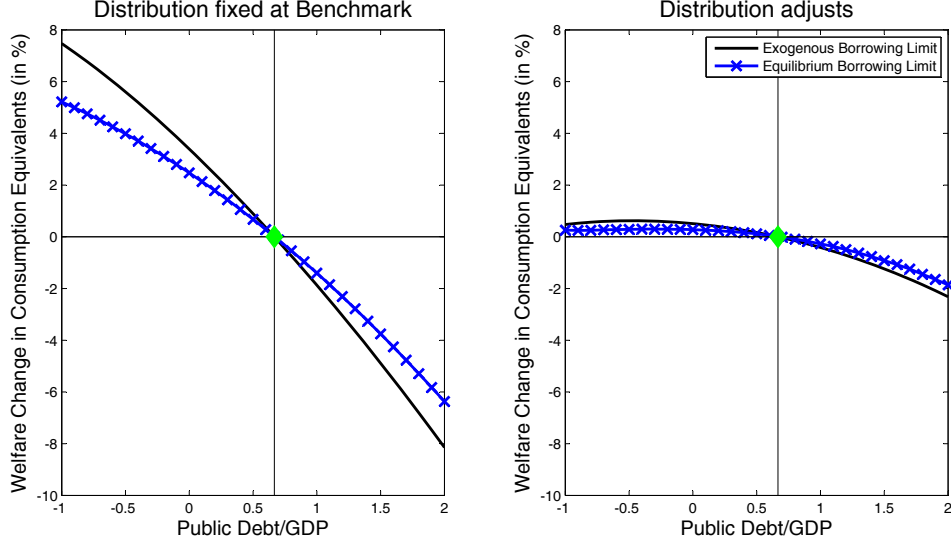


Figure 7: **Comparison of Utilitarian Welfare under Equilibrium Distribution and with Distribution fixed at Benchmark.** Here we compare the Utilitarian welfare measure (computed as the change with respect to the benchmark in consumption equivalent terms) implied by our model when the distribution over assets and income states is fixed at the benchmark (left-hand side panel) with the welfare measure with the equilibrium distribution (right-hand side panel) (see also Figure 5). We show these welfare measures for different stationary equilibria that differ with respect to the public debt/GDP ratio. In the benchmark public debt amounts to 2/3 of GDP. The labor income tax is adjusted to balance the budget. Two cases: (i) exogenous (fixed) borrowing limit (black line); (ii) equilibrium borrowing limit (blue line with crosses).

reduces welfare.²²

In our model with equilibrium borrowing limits, there are two additional complications. First, as noted above, the equilibrium borrowing constraint changes over the transition path. Second, the short-run welfare effects from changes in the private borrowing limit can be very different from their long-run impact.²³ In a partial equilibrium model, Fulford (2013) shows that the welfare effects of relaxing borrowing constraints are positive in the short-run, but might be negative in the long-run. In sum, because of these caveats, it is difficult to directly derive policy recommendations from our findings.

6 Conclusion and Further Research

In this paper, we studied to what extent government debt is private liquidity. We generated private borrowing constraints by assuming that private loan contracts are not enforceable. Rational lenders set borrowing limits such that households do not have an incentive to default in equilibrium. Borrowing limits are therefore determined as an equilibrium outcome and depend on households' incentive to default. We showed that the equilibrium borrowing constraints are binding. Hence, changes in public

²²In Röhrs and Winter (2014), we show that the size of the transitional costs depend on details of the tax path, i.e. whether taxes are increasing or decreasing over the transition. We find that costs can be minimized by taxing wealth on surprise, such that households cannot respond. But even in this case, the transitional costs are substantial, due to redistributive effects stemming from aggregate price changes.

²³We are grateful to an anonymous referee for mentioning this point.

debt are non-neutral, and an increase in the debt/GDP leads to a higher interest rate. This in turn causes the cost of debt service to rise, which makes default more likely. As a consequence, when comparing stationary equilibria associated with different debt/GDP ratios, we found that private borrowing limits are tighter for equilibria with higher debt/GDP ratios.

We also illustrated that, relative to a model with an ad-hoc borrowing constraint, which is invariant to public policy, government debt is less efficient at relaxing private borrowing constraints in our framework, due to the endogenous response of the equilibrium constraint to a raise in public debt. However, compared to a model in which the debt limit is given by the natural borrowing constraint, we showed that government debt remains more effective at relaxing private borrowing constraints.

Our quantitative analysis also revealed that the effects of changes in government debt for welfare and real economic activity differ substantially, depending on the specification of the borrowing constraint. More specifically, if borrowing constraints are determined in equilibrium, the welfare effects of government debt are muted, precisely because public debt becomes less effective at relaxing borrowing constraints of private households.

We would like to conclude by mentioning possible directions for further research. First of all, it would be interesting to study the interaction between government debt and borrowing limits in a quantitative general equilibrium model in which borrowing constraints are determined by other types of agency problems, e.g. asymmetric information. In a stylized partial equilibrium setting, in which asymmetric information between lenders and borrowers may create adverse selection problems, Hayashi (1985) and Yotsuzuka (1987) present several examples which show that public debt has non-neutral effects and is therefore private liquidity. As long as Ricardian equivalence fails and government debt has an impact on aggregate prices, in particular on the interest rate, we would expect that an increase in government debt makes adverse selection more severe. Put differently, as long as changes in government debt lead to general equilibrium effects, more government debt should lead to tighter agency problems, independently of whether problem arises because of limited contract enforcement (as in our context) or asymmetric information. The development of general equilibrium models with asymmetric information that produce realistic outcomes regarding consumption risk sharing is ongoing, see Attanasio and Weber (2010) for a survey and Broer, Kapiča, and Klein (2013) for a recent contribution.

Moreover, as already mentioned in the introduction, our focus was exclusively on unsecured credit, which is used by households to self-insure against adverse productivity shocks. We abstracted from the collateral role that government bonds play in some sectors, such as the interbanking market (there in particular in repo transactions), therefore implicitly assuming that this channel is insignificant. Of course, this does not need to be the case. Relaxing this assumption and analyzing the total effect of public debt for private borrowing conditions is an interesting avenue for future research.

Furthermore, we concentrated on a closed economy set-up, assuming that aggregate prices are determined endogenously within the economy. Depending on the degree to which capital markets are integrated, the response of aggregate prices to changes in government debt might be substantially muted, see e.g. Azzimonti, de Francisco, and Quadrini (2014). Several facts (e.g. the home bias in portfolio holdings, which applies to both bonds and equity, see Tesar and Werner 1995) suggest that capital markets are far from being perfectly integrated. This is supported by the findings in Laubach (2009), which show that an increase in the government debt/GDP ratio indeed has a positive impact

on the interest rate in the US. Note that in our quantitative model, it turned out that the response of the interest rate was smaller than the long-run estimates presented in Laubach (2009). Therefore, the response of the equilibrium borrowing constraint to a change in public debt implied by our model appears to be at the lower end of what seems empirically plausible.

We also abstracted from aggregate risk. In a framework with both (uninsurable) idiosyncratic and aggregate risk, public debt does not only influence effective borrowing limits of private households, but also the ability of the government to smooth taxes over time. This trade-off is studied by Shin (2006) and Angeletos et al. (2012). By adding aggregate risk to our framework, this trade-off could be studied in a model in which borrowing limits for unsecured credit arise from limited enforcement problems.

Acknowledgments

We would like to thank Árpád Ábráhám, Caroline Chuard, Michael Golosov, Leo Kaas, Victor Tsyrennikov, Víctor Ríos-Rull, Fabrizio Zilibotti and participants at the 2014 Cologne Macro Workshop, the 2014 EUI Alumni Workshop in Economics, the 2014 Mannheim Workshop in Quantitative Macroeconomics and the 2014 Annual Meeting of the VfS, in particular our discussants Jochen Mankart and Stefan Niemann, for many helpful comments.

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Appendix

A: Saving in Autarky

The purpose of this part of the Appendix is to discuss to what extent allowing for saving after default changes the link between government debt and equilibrium borrowing constraints. We discuss this modification in the framework of our simplified model from Section 2.1. This is done in order to keep the notation simple. All of our findings would also hold in the richer environment of our quantitative economy.

We distinguish two cases. In the first case, households that defaulted can accumulate assets in all periods, including the period when default actually takes place. In the second case, saving in the default period is not possible.

Arguably, the second case is more in line with actual bankruptcy laws, at least in the US. However, the first case needly demonstrates that allowing for saving after default has a strong impact on the level of equilibrium borrowing constraints. All other things equal, they become tighter, compared to the benchmark without saving. Moreover, we also illustrate that public debt (via changes in the interest rate) has a stronger effect on the relative tightening of the borrowing constraints, again compared to the benchmark.

Preventing households to keep assets in the period in which they default somewhat weakens these effects. However, we are able to show that the response of equilibrium debt limits is (at least) as large as in the benchmark scenario without the option to save after default.

Saving in all periods. The recursive problem for households that defaulted on their debt reads as follows:

$$\begin{aligned} V^a(a, \epsilon; \theta) &= \max_{c, a'} \left\{ u(c) + \beta \sum_{\epsilon'} \pi(\epsilon' | \epsilon) V^a(a', \epsilon'; \theta) \right\} \\ \text{s.t. } c + a' &= (1 - \lambda)w(\theta)\epsilon + (1 + r(\theta))a + \chi \\ a' &\geq 0 \end{aligned} \tag{11}$$

The recursive problem of those households honoring their contract continues to be given as specified in problem 5. Let us define the borrowing limit by $a' \geq \xi^a(\epsilon'; \theta)$. The borrowing limit is determined in system 6, using the equilibrium default threshold $\underline{a}^a(\epsilon; \theta)$.

This equilibrium default threshold has the same properties that were established in Proposition 1 and Corollary 1 for the benchmark case, as long as the default penalty λ is non-negative.

Proposition 3. *Suppose that the default penalty $\lambda \geq 0$. Under Assumption 1, condition (6) defines a unique and finite default threshold $\underline{a}^a(\epsilon; \theta)$ for every ϵ and θ if the households' problem after default is given by (11). Moreover, $\underline{a}^{nat} < \underline{a}^a(\epsilon; \theta) \leq 0$.*

The following Proposition discusses the special case in which the default penalty λ is assumed to be zero. In this case, the autarky option is so attractive that no borrowing can be sustained in equilibrium. This illustrates how facilitating self-insurance after default by introducing saving leads to a tighter equilibrium borrowing limit.

Proposition 4. *Suppose that $\lambda = 0$. Then, $\underline{a}^a(\epsilon; \theta) = 0$ is the unique equilibrium default threshold.*

From Proposition 3 and Proposition 4 together, we can infer that the unique equilibrium default threshold $\underline{a}^a(\epsilon; \theta)$ will only be negative if the default penalty λ is positive. Hence, compared to the benchmark case where saving in autarky is not possible, a higher default penalty will be required in order to achieve the same amount of borrowing.

Whereas the previous two propositions presented statements about the level of the equilibrium borrowing constraint, the following proposition refers to the response of the debt limit to a change in the interest rate, keeping all other parameters constant.

Proposition 5. *Suppose that there is an equilibrium distribution $\theta(a, \epsilon)$ that is consistent with a stationary equilibrium as specified by Definition 1. The associated aggregate prices are then given by $r(\theta)$ and $w(\theta)$, and the lump-sum tax that clears the government's budget constraint by $\chi(\theta)$. Moreover, $\underline{a}^a(\epsilon; r(\theta), w(\theta), \chi(\theta))$ is the associated equilibrium default threshold, which is determined by condition (6).*

Now consider an interest rate $r^+ > r(\theta)$. Condition (6) together with r^+ , $w(\theta)$ and $\chi(\theta)$ imply a (partial) equilibrium default threshold $\underline{a}^+(\epsilon; r^+, w(\theta), \chi(\theta))$, such that $\underline{a}^{a+}(\epsilon; r^+, w(\theta), \chi(\theta)) \geq \underline{a}^a(\epsilon; r(\theta), w(\theta), \chi(\theta))$ for every ϵ and θ as long as the default penalty is non-negative.

Proposition 5 extends Proposition 2 to the case where households can save after default. Note that Proposition 5 does not depend on the fact that the value of default V^a is increasing in the interest rate, which will be true as long as households that defaulted actually make use of their opportunity to save. Households, in turn, will make use of the option to save, as they are subject to otherwise uninsurable productivity shocks. Because the value of participating in the financial market, W , is independent of whether we allow for saving after default or not, the tightening of the borrowing limit due to an increase in the interest rate will be more pronounced if households actually possess the option to save.

This result has important implications for the link between public debt and the equilibrium borrowing limit. Since the equilibrium borrowing limit is binding (this follows from $\underline{a}^{nat} < \underline{a}^a(\epsilon; \theta)$, as shown in Proposition 3), an increase in public debt will lead to a higher interest rate. Hence, all other things equal, borrowing constraints will respond more to a change in government debt if households have the option to save after default. Therefore, public debt becomes less efficient at relaxing private debt limits, if households can accumulate savings after default.

In the following, we discuss a modification of problem 11, in which households are not allowed to save in the period in which they default.

Saving after default, but not during default. An important feature of the bankruptcy code in many countries, including the US, is that households filing for bankruptcy are not permitted to keep assets beyond a maximum level. In the US, this maximum level varies across states. Following Quadrini and Ríos-Rull (2014), we approximate this exemption level by zero. The optimization problem of households that defaults then reads as

$$\begin{aligned}
V^{ar}(a = 0, \epsilon, h = 1; \theta) &= \max_c \left\{ u((1 - \lambda)w(\theta)\epsilon + \chi) + \beta \sum_{\epsilon'} \pi(\epsilon'|\epsilon) V^{ar}(a' = 0, \epsilon', h = 0; \theta) \right\} \text{ if } h = 1 \\
V^{ar}(a, \epsilon, h = 0; \theta) &= \max_{c, a'} \left\{ u(c) + \beta \sum_{\epsilon'} \pi(\epsilon'|\epsilon) V^{ar}(a', \epsilon', h = 0; \theta) \right\} \text{ if } h = 0 \\
\text{s.t. } c + a' &= (1 - \lambda)w(\theta)\epsilon + (1 + r(\theta))a + \chi \\
a' &\geq 0
\end{aligned} \tag{12}$$

The argument h in the households' problem denotes the default history. If $h = 1$, the household is defaulting. All debt is discharged and no assets can be transferred to the next period. We label the equilibrium default threshold that emerges if we use $V^{ar}(a = 0, \epsilon, h = 1; \theta)$ in condition (6) as $\underline{a}^{ar}(\epsilon; \theta)$. It can be shown that Proposition 3 continues to hold. Therefore, we know that $\underline{a}^{ar}(\epsilon; \theta)$ is unique, non-positive and finite. However, when households are not allowed to keep assets in the period in which they default, Proposition 4 does not apply anymore. Instead, we can state the following Lemma:

Lemma 1. *For a given finite default penalty λ , $V^a(a = 0, \epsilon; \theta) \geq V^{ar}(a = 0, \epsilon; \theta) \geq V(\epsilon; \theta)$ for all ϵ and θ . Moreover, $\underline{a}^a(\epsilon; \theta) \geq \underline{a}^{ar}(\epsilon; \theta) \geq \underline{a}(\epsilon; \theta)$*

Lemma 1 implies that borrowing limits are at least as tight as in the benchmark scenario, where saving after default is not allowed at all. An equivalent Proposition to Proposition 5 can be stated for this modification. Moreover, note that the same arguments can be applied as above, showing that the equilibrium borrowing limits will actually be more responsive to changes in the interest rate (and therefore also to changes in public debt) as long as households actually accumulate positive savings after default. Again, this implies that government debt is less good at relaxing private borrowing limits, relative to the benchmark case where saving in autarky was prohibited. Hence, we conclude that adding saving in autarky makes the trade-off between public debt and private debt more severe, all other things equal.

B: Proofs

This part contains all proofs.

Section 2.1, Corollary 1:

Proof. The statement that $\underline{a}(\epsilon; \theta) \leq 0$ follows from Proposition 1. Under Assumptions 1 and 2, the statement that $\underline{a}^{nat} < \underline{a}(\epsilon; \theta)$ follows from the fact that $W(a, \epsilon; \theta)$ approaches minus infinity as a goes to the natural borrowing limit \underline{a}^{nat} , while $V(\epsilon; \theta)$ is finite for all ϵ and K bounded away from zero, where $\epsilon > 0$ by assumption and $K > 0$ has to hold in equilibrium. The fact that $\underline{a}^{nat} < \underline{a}(\epsilon; \theta)$ implies that $\underline{a}(\epsilon; \theta)$ is binding for a positive fraction of the population, since under Assumptions 1 and 2, \underline{a}^{nat} is the tightest borrowing limit which is not binding. \square

Section 2.1, Proposition 2:

Proof. From Proposition 1, we know that the unique $\underline{a}(\epsilon; r(\theta), w(\theta), \chi(\theta))$ is non-positive. For households with $a = \underline{a}(\epsilon; r(\theta), w(\theta), \chi(\theta))$, this implies that their budget set from problem (5) is (weakly)

decreasing in r (strictly if $\underline{a}(\epsilon; r(\theta), w(\theta), \chi(\theta)) < 0$). Since $u(\cdot)$ is increasing in consumption, this means that $W(\underline{a}(\epsilon; r(\theta), w(\theta), \chi(\theta)), \epsilon; \theta)$ is (weakly) decreasing in r (strictly if $\underline{a}(\epsilon; r(\theta), w(\theta), \chi(\theta)) < 0$). Therefore, $W(\underline{a}(\epsilon; r(\theta), w(\theta), \chi(\theta)), \epsilon; r(\theta), w(\theta), \chi(\theta)) \geq W(\underline{a}(\epsilon; r(\theta), w(\theta), \chi(\theta)), \epsilon; r^+, w(\theta), \chi(\theta))$. Because $V(\epsilon; \theta)$ is independent of r by construction, this implies that $\underline{a}^+(\epsilon; r^+, w(\theta), \chi(\theta))$ at which $W(\underline{a}^+(\epsilon; r^+, w(\theta), \chi(\theta)), \epsilon; r^+, w(\theta), \chi(\theta)) = V(\epsilon; \theta)$ is increasing in r , because $W(a, \epsilon; \theta)$ is increasing in a , again from $u'(c) > 0$. Therefore, $\underline{a}^+(\epsilon; r^+, w(\theta), \chi(\theta)) \geq \underline{a}(\epsilon; r(\theta), w(\theta), \chi(\theta))$ for every ϵ and θ \square

Appendix A, Proposition 3:

Proof. This Proposition is an extension of Proposition 2.1 in Ábrahám and Cárceles-Poveda (2010). First, notice that the instantaneous utility function $u(\cdot)$ is continuous in consumption. Therefore, $W(a, \epsilon; \theta)$ is continuous in a . Moreover, $W(a, \epsilon; \theta)$ is also increasing in a , since $u(\cdot)$ is increasing in consumption and the budget set is monotone since everything that is feasible under $(a, \epsilon; \theta)$ is also feasible under $(a^+, \epsilon; \theta)$, where $a^+ \geq a$. Hence, $W(a^+, \epsilon; \theta) \geq W(a, \epsilon; \theta)$.

Second, from Assumption 1 (u is unbounded below), we know that $\lim_{a \rightarrow \underline{a}^{nat}} = -\infty$. Moreover, $V^a(a, \epsilon; \theta) > -\infty$. Therefore, from the fact that $\underline{a}^a(\epsilon; \theta)$ is determined such that $W(\underline{a}^a(\epsilon; \theta), \epsilon; \theta) = V^a(a = 0, \epsilon; \theta)$, it follows that $-\infty < \underline{a}^{nat} < \underline{a}^a(\epsilon; \theta)$.

Third, we can show that $\underline{a}^a(\epsilon; \theta) \leq 0$ as long as $\lambda \geq 0$. Suppose that $\underline{a}^a(\epsilon; \theta) > 0$. Let us define this borrowing limit as $\underline{a}^a(\epsilon; \theta)^{**}$. Under this constraint, let $W^{**}(a, \epsilon; \theta)$ be the solution to the non-defaulting households' problem given in (5). We will show that $\underline{a}^a(\epsilon; \theta) > 0$ is inconsistent with condition (6), the borrowing limit has to be such that

$$W^{**}(a = \underline{a}^a(\epsilon; \theta)^{**}, \epsilon; \theta) = V^a(a = 0, \epsilon; \theta) \quad (13)$$

In the following, we assume that $\lambda = 0$. The case $\lambda > 0$ follows immediately and is discussed below. Let $a'(a = 0, \epsilon; \theta)^d \geq 0$ be the optimal savings policy that solves problem (11) for a household who just defaulted, i.e. has zero assets. We now argue that this policy is also feasible for a household that honors the debt contract. However, this household can achieve more consumption (and therefore also more utility) by following the same savings rule. The consumption of the non-defaulting household is $c^h = w\epsilon + (1+r)\underline{a}^a(\epsilon; \theta)^{**} - a'(a = 0, \epsilon; \theta)^d - \underline{a}^a(\epsilon; \theta)^{**}$. Notice that non-defaulting households are forced to save at least $\underline{a}^a(\epsilon; \theta)^{**} > 0$. Consumption for the defaulting household is given by $c^d = w\epsilon - a'(a = 0, \epsilon; \theta)^d$, where $a'(a = 0, \epsilon; \theta)^d$ has to be such that $c^d > 0$, by Assumption 1. Moreover, $c^h > c^d$ because $\underline{a}^a(\epsilon; \theta)^{**} > 0$ by assumption and $r > 0$ in equilibrium. Therefore, $c^h > c^d > 0$.

In the following period, assets of households in autarky are given by $a'(a = 0, \epsilon; \theta)^d$, whereas non-defaulting households own $a'(a = 0, \epsilon; \theta)^d + \underline{a}^a(\epsilon; \theta)^{**}$. Let the optimal choice of households in autarky be $a''(a = a'(a = 0, \epsilon; \theta)^d, \epsilon; \theta)^d \geq 0$. Hence, $c'^d = w\epsilon' + (1+r)a'(a = 0, \epsilon; \theta)^d - a''(a = a'(a = 0, \epsilon; \theta)^d, \epsilon; \theta)^d$ and $c'^d > 0$. Consumption of non-defaulting households following the same rule but obeying their asset constraint is given by $c'^h = w\epsilon' + (1+r)(a'(a = 0, \epsilon; \theta)^d + \underline{a}^a(\epsilon; \theta)^{**}) - a''(a = a'(a = 0, \epsilon; \theta)^d, \epsilon; \theta)^d - \underline{a}^a(\epsilon; \theta)^{**}$, which implies that $c'^h > c'^d > 0$. The same reasoning can be applied to all other periods as well. Non-defaulting households can consume more in all periods because of their higher initial assets, which would be lost in case of default.

Hence, $W^{**}(a = \underline{a}^a(\epsilon; \theta)^{**}, \epsilon; \theta) > V^a(a = 0; \theta)$, which contradicts condition (13) for all ϵ and θ .²⁴

²⁴Recall that we are looking for the loosest possible default threshold. Of course, because $W^{**}(a = \underline{a}^a(\epsilon; \theta)^{**}, \epsilon; \theta) > V^a(a = 0; \theta)$, $\underline{a}^a(\epsilon; \theta)^{**} > 0$ also prevents default.

If $\lambda > 0$, $V^a(\epsilon, a = 0; \theta)$ becomes even smaller, all other things equal, which reinforces the contradiction. Therefore, we conclude that $\underline{a}^a(\epsilon; \theta) \leq 0$.

The uniqueness of $\underline{a}^a(\epsilon; \theta)$ then follows from the fact that W is increasing in a , while V is independent of a as long as we are considering $a \leq 0$. \square

Appendix A, Proposition 4:

Proof. We distinguish three cases: $\underline{a}^a(\epsilon; \theta) < 0$, $\underline{a}^a(\epsilon; \theta) > 0$ and $\underline{a}^a(\epsilon; \theta) = 0$. We show that the first two cases contradict condition (6).

1. Suppose that $\underline{a}^a(\epsilon; \theta) < 0$. Call this borrowing limit $\underline{a}^a(\epsilon; \theta)^*$. Under this constraint, let $W^*(a, \epsilon; \theta)$ be the solution to the non-defaulting households' problem given in (5). By condition (6), this implies that

$$W^*(a = \underline{a}^a(\epsilon; \theta)^*, \epsilon; \theta) = V^a(a = 0, \epsilon; \theta) \quad (14)$$

Let $a'(\underline{a}^a(\epsilon; \theta)^*, \epsilon; \theta)^h \geq \underline{a}^a(\epsilon'; \theta)^*$ be the optimal savings decision of non-defaulting households at the borrowing constraint. The strategy of the proof by contradiction is to show that this savings decision is feasible for defaulting households as well. Implementing it in autarky leads to more consumption (and therefore also utility), which contradicts condition (14).

Notice that consumption when honoring the implicit debt contract is $c^h = w\epsilon + (1+r)\underline{a}^a(\epsilon; \theta)^* - a'(\underline{a}^a(\epsilon; \theta)^*, \epsilon; \theta)$. A defaulting household has more resources, as no debt needs to be repaid. These additional resources are saved, such that $a'(\underline{a}^a(\epsilon; \theta)^*, \epsilon; \theta)^d = a'(\underline{a}^a(\epsilon; \theta)^*, \epsilon; \theta)^h - \underline{a}^a(\epsilon; \theta)^*$. Consumption in case of default is then given by $c^d = w\epsilon + \underline{a}^a(\epsilon; \theta)^* - a'(\underline{a}^a(\epsilon; \theta)^*, \epsilon; \theta)^h - \underline{a}^a(\epsilon; \theta)^*$. By Assumption 1, $a'(\underline{a}^a(\epsilon; \theta)^*, \epsilon; \theta)$ has to be such that $c^h > 0$. This implies that also $c^d > 0$. Moreover, $c^d > c^h$ since $r > 0$ in equilibrium and $\underline{a}^a(\epsilon; \theta)^* < 0$ by assumption.

Let us consider the next period. At the beginning of the next period, the non-defaulting household owns $a'(\underline{a}^a(\epsilon; \theta)^*, \epsilon; \theta)$, and the savings decision for the following period is therefore

$$a''(a'(\underline{a}^a(\epsilon; \theta)^*, \epsilon; \theta), \epsilon'; \theta)^h \geq \underline{a}^a(\epsilon''; \theta)^*$$

Consequently, $c'^h = w\epsilon' + (1+r)a'(\underline{a}^a(\epsilon; \theta)^*, \epsilon; \theta) - a''(a'(\underline{a}^a(\epsilon; \theta)^*, \epsilon; \theta), \epsilon'; \theta)^h$. For the household that defaulted in the previous period, we get $c'^d = w\epsilon' + (1+r)(a'(\underline{a}^a(\epsilon; \theta)^*, \epsilon; \theta) - \underline{a}^a(\epsilon; \theta)^*) - a''(a'(\underline{a}^a(\epsilon; \theta)^*, \epsilon; \theta), \epsilon'; \theta)^h - \underline{a}^a(\epsilon; \theta)^*$. Therefore, $c'^d > c'^h$ as long as $r > 0$. The same logic applies to all future periods as well. Defaulting households can sustain a higher consumption path because they receive positive interest payments on their additional savings, which they obtain by not paying back debt. These additional savings also compensate for the lack of borrowing opportunities in autarky.

Hence, we conclude that $W^*(a = \underline{a}^a(\epsilon'; \theta)^*, \epsilon; \theta) < V^a(\epsilon, a = 0; \theta)$ for all ϵ and θ , which contradicts condition (14). Therefore, $\underline{a}^a(\epsilon'; \theta)^*$ cannot be the equilibrium default threshold.

2. Suppose that $\underline{a}^a(\epsilon; \theta) > 0$. Let us define this borrowing limit as $\underline{a}^a(\epsilon; \theta)^{**}$. Under this constraint, let $W^{**}(a, \epsilon; \theta)$ be the solution to the non-defaulting households' problem given in (5). By condition (6), the borrowing limit has to be such that

$$W^{**}(a = \underline{a}^a(\epsilon; \theta)^{**}, \epsilon; \theta) = V^a(a = 0, \epsilon; \theta) \quad (15)$$

Let $a'(a = 0, \epsilon; \theta)^d \geq 0$ be the optimal savings policy that solves problem (11) for a household who just defaulted, i.e. has zero assets. We now argue that this policy is also feasible for a household that honors the debt contract. However, this household can achieve more consumption (and therefore also more utility) by following the same savings rule. The consumption of the non-defaulting household is $c^h = w\epsilon + (1+r)\underline{a}^a(\epsilon; \theta)^{**} - a'(a = 0, \epsilon; \theta)^d - \underline{a}^a(\epsilon; \theta)^{**}$. Notice that non-defaulting households are forced to save at least $\underline{a}^a(\epsilon; \theta)^{**} > 0$. Consumption for the defaulting household is given by $c^d = w\epsilon - a'(a = 0, \epsilon; \theta)^d$, where $a'(a = 0, \epsilon; \theta)^d$ has to be such that $c^d > 0$, by Assumption 1. Moreover, $c^h > c^d$ because $\underline{a}^a(\epsilon; \theta)^{**} > 0$ by assumption and $r > 0$ in equilibrium. Therefore, $c^h > c^d > 0$.

In the following period, assets of households in autarky are given by $a'(a = 0, \epsilon; \theta)^d$, whereas non-defaulting households own $a'(a = 0, \epsilon; \theta)^d + \underline{a}^a(\epsilon; \theta)^{**}$. Let the optimal choice of households in autarky be $a''(a = a'(a = 0, \epsilon; \theta)^d, \epsilon; \theta)^d \geq 0$. Hence, $c'^d = w\epsilon' + (1+r)a'(a = 0, \epsilon; \theta)^d - a''(a = a'(a = 0, \epsilon; \theta)^d, \epsilon; \theta)^d$ and $c'^d > 0$. Consumption of non-defaulting households following the same rule but obeying their asset constraint is given by $c'^h = w\epsilon' + (1+r)(a'(a = 0, \epsilon; \theta)^d + \underline{a}^a(\epsilon; \theta)^{**}) - a''(a = a'(a = 0, \epsilon; \theta)^d, \epsilon; \theta)^d - \underline{a}^a(\epsilon; \theta)^{**}$, which implies that $c'^h > c'^d > 0$. The same reasoning can be applied to all other periods as well. Non-defaulting households can consume more in all periods because of their higher initial assets, which would be lost in case of default.

Hence, $W^{**}(a = \underline{a}^a(\epsilon; \theta)^{**}, \epsilon; \theta) > V^a(\epsilon, a = 0; \theta)$, which contradicts condition (15) for all ϵ and θ .²⁵

3. We now turn to the case $\underline{a}^a(\epsilon; \theta) = 0$. We label this borrowing limit as $\underline{a}^a(\epsilon; \theta)^{***}$. The solution of problem (5) under this constraint is labeled as $W^{***}(a = 0, \epsilon; \theta)$. Notice that $W^{***}(a = 0, \epsilon; \theta) = V^a(a = 0, \epsilon; \theta)$ since both defaulting and non-defaulting households are then subject to the same borrowing limit.

□

Appendix A, Proposition 5:

Proof. From Proposition 3, we know that the unique $\underline{a}^a(\epsilon; r(\theta), w(\theta), \chi(\theta))$ is non-positive. For households with $a = \underline{a}^a(\epsilon; r(\theta), w(\theta), \chi(\theta))$, this implies that their budget set from problem (5) is (weakly) decreasing in r (strictly if $\underline{a}^a(\epsilon; r(\theta), w(\theta), \chi(\theta)) < 0$). Since $u(\cdot)$ is increasing in consumption, this means that $W(\underline{a}^a(\epsilon; r(\theta), w(\theta), \chi(\theta)), \epsilon; \theta)$ is (weakly) decreasing in r (strictly if $\underline{a}^a(\epsilon; r(\theta), w(\theta), \chi(\theta)) < 0$). Therefore, $W(\underline{a}^a(\epsilon; r(\theta), w(\theta), \chi(\theta)), \epsilon; r(\theta), w(\theta), \chi(\theta)) \geq W(\underline{a}^a(\epsilon; r(\theta), w(\theta), \chi(\theta)), \epsilon; r^+, w(\theta), \chi(\theta))$. Because $V(\epsilon; \theta)$ is independent of r by construction, this implies that $\underline{a}^{a+}(\epsilon; r^+, w(\theta), \chi(\theta))$ at which $W(\underline{a}^{a+}(\epsilon; r^+, w(\theta), \chi(\theta)), \epsilon; r^+, w(\theta), \chi(\theta)) = V(\epsilon; \theta)$ is increasing in r , because $W(a, \epsilon; \theta)$ is increasing in a , again from $u'(c) > 0$. Therefore, $\underline{a}^{a+}(\epsilon; r^+, w(\theta), \chi(\theta)) \geq \underline{a}^a(\epsilon; r(\theta), w(\theta), \chi(\theta))$ for every ϵ and θ

□

²⁵Recall that we are looking for the loosest possible default threshold. Of course, because $W^{**}(a = \underline{a}^a(\epsilon; \theta)^{**}, \epsilon; \theta) > V^a(\epsilon, a = 0; \theta)$, $\underline{a}^a(\epsilon; \theta)^{**} > 0$ also prevents default.

Appendix A, Lemma 1:

Proof. Comparing V^a and V^{ar} , households can always choose not save in the first period after default if this is optimal, therefore $V^a(a = 0, \epsilon; \theta) \geq V^{ar}(a = 0, \epsilon; \theta)$ for all ϵ and θ . The same logic applies to all future periods as well, therefore $V^{ar}(a = 0, \epsilon; \theta) \geq V(\epsilon; \theta)$ for all ϵ and θ . The fact that $\underline{a}^a(\epsilon; \theta) \geq \underline{a}^{ar}(\epsilon; \theta) \geq \underline{a}(\epsilon; \theta)$ then follows from using the fact $V^a(a = 0, \epsilon; \theta) \geq V^{ar}(a = 0, \epsilon; \theta) \geq V(\epsilon; \theta)$ in condition (6), since the value of not defaulting, W , is independent of the specification of the autarky problem. \square

C: Detrended Formulation of the Households' Maximization Problem

In our model, there is a balanced growth path along which variables will be growing at the rate of technology growth. To find the stationary equilibrium of the model, we first have to detrend variables with respect to this exogenous productivity growth component to obtain a formulation where variables are constant in the balanced growth equilibrium. This procedure was also used in the earlier literature, for example by Aiyagari and McGrattan (1998) and Flodén (2001). Denote a detrended variable by "tilde": $\tilde{x} = \frac{x}{Y}$. The present value of lifetime utility can then be denoted as follows:

$$U(\{\tilde{c}_t\}_{t=1,2,\dots}, \{1-l_t\}_{t=1,2,\dots}) = E_0 \sum_{t=0}^{\infty} \beta^t Y_t^{\eta(1-\mu)} u(\tilde{c}_t, 1-l_t)$$

Now using the fact that $Y_t = Y_0(1+g)^t$, where Y_0 is output in period 0, we can write:

$$\begin{aligned} U(\{\tilde{c}_t\}_{t=1,2,\dots}, \{1-l_t\}_{t=1,2,\dots}) &= Y_0^{\eta(1-\mu)} E_0 \sum_{t=0}^{\infty} \beta^t (1+g)^{t\eta(1-\mu)} u(\tilde{c}_t, 1-l_t) \\ &= Y_0^{\eta(1-\mu)} E_0 \sum_{t=0}^{\infty} \tilde{\beta}^t u(\tilde{c}_t, 1-l_t) \end{aligned}$$

where $\tilde{\beta} = \beta \cdot (1+g)^{\eta(1-\mu)}$.

Similarly, we can find a detrended version of the household budget constraint by dividing it by Y_t :

$$\begin{aligned} \frac{c_t}{Y_t} + \frac{Y_{t+1}}{Y_t} \frac{a_{t+1}}{Y_{t+1}} &= \frac{\bar{w}_t}{Y_t} \epsilon l_t + (1 + \bar{r}_t) \frac{a}{Y_t} - \frac{\chi}{Y_t} \\ \tilde{c}_t + (1+g)\tilde{a}_{t+1} &= \tilde{w}_t \epsilon l_t + (1 + \bar{r}_t) \tilde{a}_t - \tilde{\chi} \end{aligned}$$

Also the borrowing constraint can be detrended:

$$\tilde{a}_{t+1} \geq \tilde{a}_t$$

Consequently, the detrended version of (8) is given by:

$$\begin{aligned} W(\tilde{a}, \epsilon; \theta) &= \max_{\tilde{a}', \tilde{c}, l} Y_0^{\eta(1-\mu)} \left\{ u(\tilde{c}, 1-l) + \tilde{\beta} \sum_{\epsilon'} \pi(\epsilon'|\epsilon) W(\tilde{a}', \epsilon'; \theta) \right\} \\ \text{s.t. } \tilde{c} + (1+g)\tilde{a}' &= \tilde{w}\epsilon l + (1 + \bar{r})\tilde{a} - \tilde{\chi} \\ \tilde{a}' &\geq \tilde{a} \\ \theta &= \Gamma[\theta] \end{aligned}$$

The detrended version of (7) is straightforward and therefore omitted.

D: Additional results: Asset income tax instead of labor income tax is adjusted in order to balance the government's budget constraint (not for publication)

In this section, we present the results of a robustness exercise, in which we adjust the asset income tax in order to balance the government's budget constraint. In the main text, we kept the asset income tax constant and instead changed the labor income tax. It turns out that our main results are robust to a change in the tax instrument.

In order to facilitate a comparison between the results, we present the Figures in the same order as in the main text.

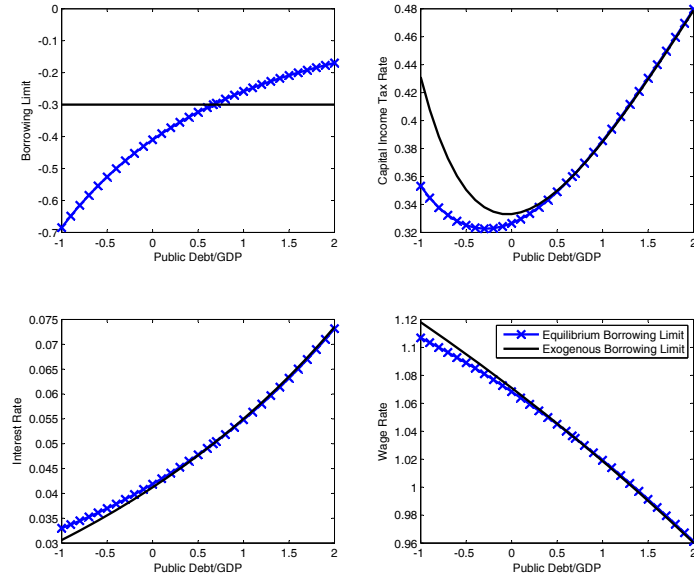


Figure 8: Implied Borrowing Limits, Tax Rates, Interest Rates and Wage Rates for Different Stationary Equilibria. In this exercise we plot the changes in the borrowing limits (first panel), tax rates (second panel), interest rates (third panel) and wage rates (fourth panel) implied by our model for different stationary equilibria that differ with respect to the public debt/GDP ratio. In the benchmark public debt amounts to 2/3 of GDP. The asset income tax is adjusted to balance the budget. Two cases: (i) exogenous (fixed) borrowing limit (black line); (ii) equilibrium borrowing limit (blue line with crosses).

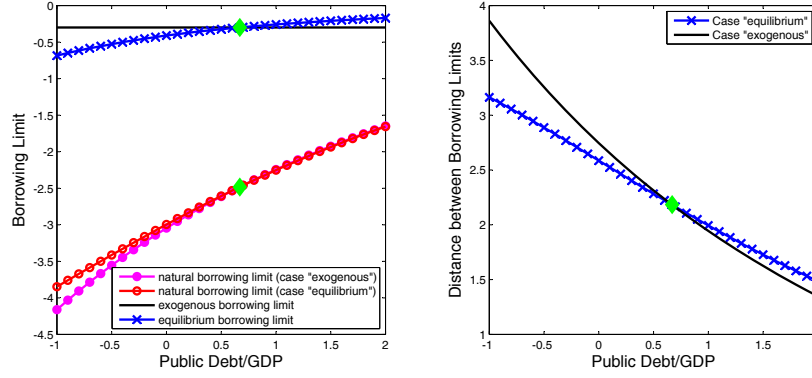


Figure 9: **Natural Borrowing Limit, Endogenous Borrowing Limit and Distance Between them.** In the left-hand side panel we plot the changes in the natural borrowing limit case 'exogenous' (pink filled circles), the natural borrowing limit case 'equilibrium' (red empty circles) the exogenous (fixed) borrowing limit (black line) and the equilibrium borrowing limit (blue crosses) arising from limited commitment for stationary equilibria differing with respect to the public debt/GDP ratio. In the right-hand side panel we plot the distance between the two borrowing limits for the same set of stationary equilibria. At the benchmark public debt amounts to 2/3 of GDP (green diamond).

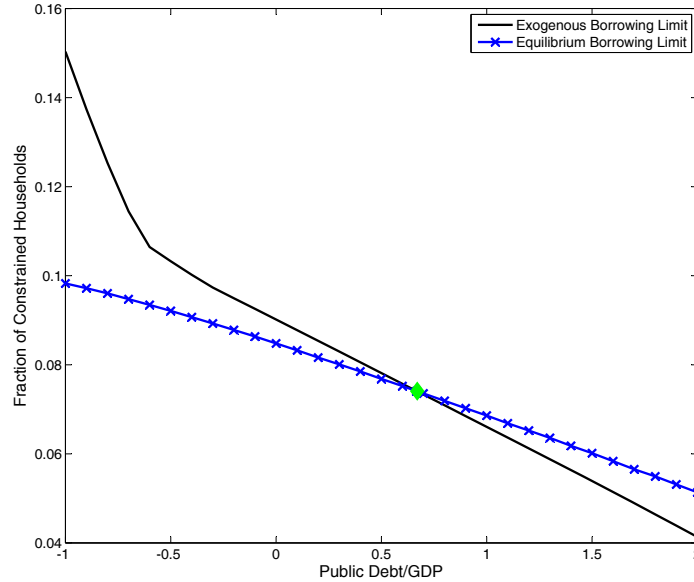


Figure 10: **Fraction of Constrained Households.** Here we plot the number of households at the borrowing constraint for stationary equilibria differing with respect to the public debt/GDP ratio. At the benchmark public debt amounts to 2/3 of GDP (green diamond). Two cases: (i) exogenous borrowing limit (black line); (ii) equilibrium borrowing limit (blue line with crosses).

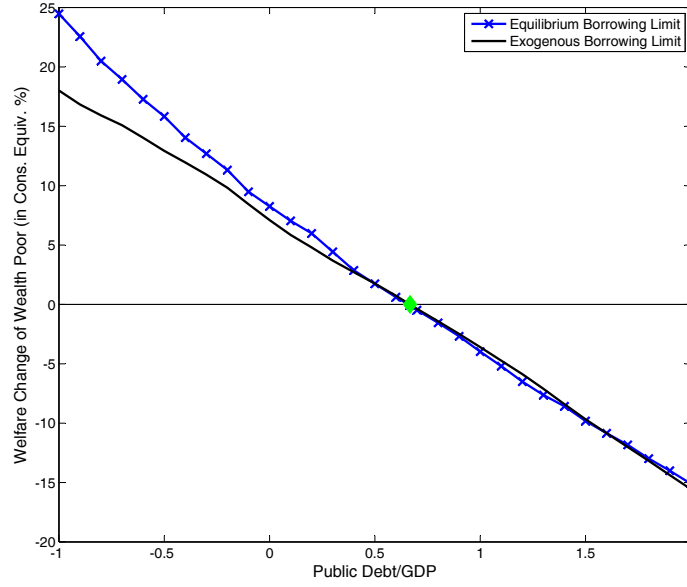


Figure 11: **Welfare of Wealth Poor.** Here we plot the change in consumption equivalent welfare of wealth poor agents holding zero or negative assets, for stationary equilibria differing with respect to the public debt/GDP ratio. At the benchmark public debt amounts to $2/3$ of GDP (green diamond). Two cases: (i) exogenous borrowing limit (black line); (ii) equilibrium borrowing limit (blue line with crosses).

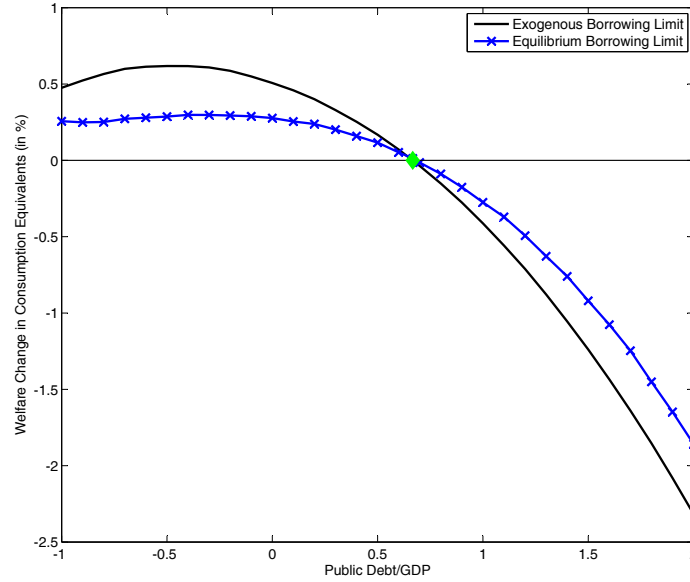


Figure 12: **Welfare of Total Population.** We plot the welfare change in consumption equivalent units (on the y-axis) for stationary equilibria differing in the public debt/GDP ratio (on the x-axis) for the total population. At the benchmark public debt amounts to $2/3$ of GDP (green diamond). Two cases: (i) exogenous borrowing limit (black line); (ii) equilibrium borrowing limit (blue line with crosses).